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Relation of fertilization rates to pasture yield and utilization

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RELATION OF FERTILIZATION RATES
TO PASTURE YIELD AND UTILIZATION

by

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I. INTRODUCTION

A. General

One central problem confronts any farmer acting in his capacity as a decision maker. It concerns planning for a future which is always uncertain in respect to yields, prices, or both. A drought may parch his land to an extent that in one year his crops are scarcely worth harvesting. In another year seeds or young plants may be washed out by excessive rain or floods. Yet, in another growing season, climatic factors may combine favorably to provide him with bumper crops. Added to this, the free market mechanism usually penalizes him in price when he and his neighbors have a large crop or livestock output. In contrast, in years of drought, his small output does not allow him to take advantage of high prices.

In practice the conditions that the farmer faces may not be as extreme as those suggested. The examples, however, emphasize that *ex ante* manipulation of resources in order to maximize future value product, presents decision making problems.

Consider decisions centering around fertilizer use. The decision of whether or not to use fertilizer might seem easy under today's scientific farming methods. Yet in 1954 only 58 percent of Iowa farmers used fertilizer. The average amount spread was only 5.5 tons per farm (21).

However, once it has been decided to use fertilizer, a whole new series of choices must be made. Ideally, a choice of specific fertilizer elements should be made on the basis of soil test data and known crop requirements. But often the former are not available for the individual farm. Thus in using an NPK fertilizer mix, as opposed to NP or PK mixtures, the farmer may be motivated by personal preference or the theory that a little of everything is beneficial. Alternatively the neighbors or the fertilizer salesman may influence him. The rate of application per acre may depend on such factors as the mechanical condition of the manure distributor or the expected crop or pasture yield. Or the deciding influence may be advice of farmer friends or extension workers.

This study seeks to demonstrate that <u>some</u> precision can be given to decisions concerning fertilizer use, even in the presence of uncertainty. Particular attention is paid to only a few of the factors that the farmer takes into account. Hence the analysis may possibly be criticized on the grounds of over simplification.

Initially the farmer decision maker is considered to be concerned only with the problem of the profit maximizing rate of fertilization of alfalfa, when one, two or three cuttings can be expected with certainty. Different capital situations are examined. The amount of money available for buying fertilizer may or may not be limited. The farmer may or may not apply fertilizer, depending on how he considers the returns compare, if the money was spent on other inputs.

The second more realistic case assumes that the farmer is uncertain as to whether he will get one, two or three cuts from his alfalfa.

The various optima are all arrived at as if they were *ex post* decisions. Unhappily, as every extension worker knows, the farmer requires *ex ante* advice. Few advisory workers would be willing to base predictions on the result of one experiment carried out under particular environmental conditions. However they would probably admit that the data would have predictive value for future experiments, if the circumstances surrounding these later experiments were similar. A series of trials on different soil types and in different locations would enable reliable recommendations to be made. These are the grounds for asserting that advice based on *ex post* information can help to reduce decision making uncertainty.

There is another aspect of fertilizer use. The quantity of fertilizer applied may be based not only on expected yields but also on the method of utilization of the crop. The farmer may sell alfalfa as hay rather than keep it for feeding purposes. If the hay is sold he may be prepared to apply a greater quantity of fertilizer, in expectation of higher profits.

Therefore the third case deals with an extension of the first (in which yield is certain). In addition it is assumed that utilization is certain. The alfalfa may be kept as hay for winter supplementary feed or it may be sold. Alternatively it may be used green-chopped for dairy cows or as summer pasture for pigs.

Throughout the growing season the value of the alfalfa in various uses may change. Thus a drought may force the farmer to feed *in situ* rather than as hay. Acquisition of labor saving machinery may reduce the cost of haymaking. This leads to the fourth case to be examined. This is essentially a yield uncertainty situation. In addition, there is uncertainty concerning the utilization of the crop, at the time of application of the fertilizer.

The analysis is relatively simplified as compared to the on-farm situation. But it does reduce some of the variable elements in decision making to more measurable terms.

II. OBJECTIVES

The previous chapter gave a general review of the area in which the study is being made.

More specifically, the objectives are: (a) to decide on the form of a production function which best expresses the relationship between P_2O_5 and K_2O application and alfalfa yield for a particular experiment; (b) to fit this function to the yield data from the experiment; (c) to attempt to derive profit maximizing fertilization rates for the alfalfa under specific circumstances. These circumstances relate to the number of cuttings and the method of utilization of the crop.

The first two aims are discussed in Chapter IV. The third objective is the major preoccupation of the study. Chapter V examines the situation in which the number of cuttings is certain. Capital available for fertilizer is first considered unlimited, then later assumed restricted. In Chapter VI profit maximization is discussed when the number of cuttings expected is uncertain. Chapter VII considers that utilization of the crop is known *ex ante*. It is shown that fertilization levels may be adjusted accordingly. In Chapter VIII both utilization and the number of cuttings are assumed unknown when the fertilizer is applied. Alternative fertilizer use decisions are examined for this situation.

In general the aim of the analysis is to show that precision can be given to fertilizer use recommendations.

III. SOURCE OF BASIC DATA

The experiment from which the data were obtained was one of a series of pasture fertilization trials. These were carried out in southeast Iowa under the direction of the Agronomy Department of Iowa State College. The trial analyzed in this study was laid down on Weller silt loam in Van Buren County in 1952. The design was a 3 x 3 factorial, replicated twice, giving a total of 18 observations. Two of these were checks. P_2O_5 and K_2O fertilizer were both applied at three levels (0, 60 and 120 pounds per acre). The alfalfa was cut three times during the course of the growing season. Three cuttings are considered normal in this area. Table 1 includes the results of these three cuttings.

Table 1. Weller silt loam, yields of alfalfa, 1952 (tons oven dry material per acre)

Rate of fertilization (lbs./acre)		1st cut Replicate		2nd cut Replicate		3rd cut Replicate	
P_2O_5	K_2O	I	II	I	II	I	II
0	0	.73	.84	.69	.95	.45	.55
60	0	.94	1.41	.85	.95	.60	.50
120	0	1.16	1.38	.83	.92	.62	.57
0	60	1.05	1.21	.85	.85	.55	.57
60	60	1.32	1.49	.92	.92	.65	.67
120	60	1.27	1.56	.97	1.16	.62	.72
0	120	1.05	1.32	.90	.92	.57	.62
60	120	1.49	1.27	1.00	1.00	.69	.57
120	120	1.68	1.38	1.07	1.00	.67	.65

IV. DERIVATION OF PRODUCTION FUNCTIONS

A. Introduction

Chapters I, II and III outlined the scope and nature of the study. The basic yield data around which the analysis is built were also presented. This chapter discusses the alternative forms of production functions which might be used. Regression equations are then presented, together with their consequent statistical tests of significance. Finally, production surfaces, isoquants and isoclines are shown and discussed.

B. Selection of a Function

There is agreement in the literature on fertilizer production functions (9, 14) about the form of function to be chosen as best representing the relationship between input and output quantities. It should be some compromise between what is biologically compatible and what is statistically sound. Less emphasis has been placed on the ease and reliability with which economically meaningful quantities can be derived from the function. A practical consideration which must be taken into account is that time and research funds are not limitless. This is true for researchers working in a non-academic environment and for some university personnel. It may be desirable for work to be pressed forward in seeking an "ideal" function characterizing response of plants to fertilizer. However, the fact remains that guidance must be given now to farmers and others.

Fortunately the choice of a suitable form of function is not as difficult as it might appear. The more generally acceptable[1] types of functions fall into three groups: (a) exponentials, (b) the power function (Cobb-Douglas) and (c) polynomials.

The most widely used exponentials are the Mitscherlich and the Spillman. But these lack elegance from the computational point of view. Also they have the disadvantage that statistical tests such as standard errors and t values cannot be computed, for the coefficients. Again the product curve for both is regarded as asymptotic to some maximum yield. This fact is not easily reconcilable with observed phenomena of negative marginal products found in some fertilizer experiments. The assumption that elasticity of response is less than 1 over all ranges of inputs may not be realistic at the lower rates of fertilization. This is especially true for an impoverished soil.

The power function presents no computational problems and is amenable to statistical tests. But the assumption of

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[1]Ruling out such special cases as the Bray modification of the Mitscherlich, Janisch's complex exponential, or Briggs' hyperbolic form.

constant elasticity of production may be justifiable only for a small range of fertilizer inputs. Relaxation of the assumptions of constant elasticity and symmetry gives a more realistic form (6). Unfortunately the computations for finding economic optima become extensive.

By contrast, polynomial models are easy to fit and test. Also they are flexible in the sense that terms may be added or dropped easily. Furthermore, no assumptions are made about the elasticity of response. Negative marginal products are allowed for by the inclusion of a squared term with (usually) negative sign. (The signs do not always work out to be negative, in which case the problem of adjusting for diminishing returns remains unsolved.) The chief criticism of such functions is that only linear and interaction terms can be justified as far as plant growth is concerned. The same cannot be said for squared or cubed terms or terms raised to some power (e.g. square root transformations). Thus inconsistencies of one sort or another can be found in all these functions seeking tocquantify input-output relationships. Eventually a choice, not based entirely on objective grounds must be made.

As far as the present study is concerned, a polynomial function with an interaction term was used. Specifically, the form of function is as follows:

$$Y = a + bP + cK - dP^2 - eK^2 + fPH \quad \text{where}$$

Y = expected yield; a = yield intercept; b, c, d and e are the regression coefficients and P and K are the fertilizer inputs.

It was decided to fit a quadratic rather than a square root function. This is in accord with the criteria used by Heady (8) in selecting between these two types of polynomials.

C. Regression Analysis

The alfalfa yields of Table 1 provide the basic data used in this study. As a foundation for the analysis which follows a regression equation has been fitted to the data of each cutting. Yields for the first and second cuttings were added together and a further function was fitted to this total. Likewise a regression equation was computed for the sum of the yields of the three cuttings.[1] The five equations presented in the order mentioned above are:

$$Y = .822235 + .007042P + .006181K - .000028P^2 - .000027K^2 - .000010PK \quad (4.1)$$

$$Y = .811947 + .001278P + .001403K - .000004P^2 - .000006K^2 + .000005PK \quad (4.2)$$

$$Y = .490277 + .001431P + .002181K - .000005P^2 - .000012K^2 - .000002PK \quad (4.3)$$

[1]Subsequent interpretation of the data in terms of hay rather than oven dry material means that the regression coefficients alter slightly.

$$Y = 1.634182 + .008320P + .007584K - .000032P^2 - .000033K^2 - .000005PK \qquad (4.4)$$

$$Y = 2.124459 + .009751P + .009765K - .000037P^2 - .000045K^2 - .000007PK \qquad (4.5)$$

Equation 4.4 could have been obtained by adding 4.1 and 4.2 and equation 4.5 by adding 4.1, 4.2 and 4.3. However as a further check on accuracy separate regressions were computed.

Table 2 includes the analyses of variance for the yield data corresponding to each of the above regression equations. In assessing whether the function chosen characterizes the data adequately, one common criterion is the size of the R^2's[1]. In Table 2 these seem satisfactory when compared with the R^2's derived from similar data (3, 10).

The overall significance of the regressions were tested by means of the F ratio (the null hypothesis is $b'y_1 = b'y_2$ etc. = 0). Of the data of most interest in future chapters, namely the first cut, the first plus second cut and the first plus second plus third cut, the F's are all significant at less than the 5 per cent level. For the second and third cuts taken by themselves, the F values fall just outside the 5 per cent level.

A further criterion suggested by Mason (14) is that the

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[1] $R^2 = \dfrac{\text{Reduction in sum of squares due to regression}}{\text{Treatment sum of squares}}$

Table 2. Weller silt loam, analyses of variance for alfalfa cuttings

	Source of variation		Degrees of freedom		Sum of squares	Mean square	F
Cutting 1	Total		17		1.090361		
(Regression 4.1)	Replicates		1		.076049		
	Treatments		8		.774711		
R^2 = .959	Due to regression	5		.743363		.148672	4.96*
R = .979	Lack of fit	3		.031348			
	Error		8		.239601	.029950	
Cutting 2	Total		17		.177694		
(Regression 4.2)	Replicates		1		.019338		
	Treatments		8		.114144		
R^2 = .863	Due to regression	5		.098539		.019708	3.57+
R = .929	Lack of fit	3		.015605			
	Error		8		.044212	.005526	
Cutting 3	Total		17		.079511		
(Regression 4.3)	Replicates		1		0		
	Treatments		8		.054211		
R^2 = .918	Due to regression	5		.049753		.009951	3.15+
R = .958	Lack of fit	3		.004458			
	Error		8		.025300	.003162	

*$P \leq .05$

+$P \doteq .05$

Table 2. (Continued)

	Source of variation		Degrees of freedom		Sum of squares	Mean square	F
Cuttings 1 + 2	Total		17		1.434600		
(Regression 4.4)	Replicates		1		.172088		
	Treatments		8		1.378500		
R^2 = .982	Due to regression	5		1.353403		.270681	5.04*
R = .991	Lack of fit	3		.025097			
	Error		8		.384012	.048001	
Cuttings 1 + 2 + 3	Total		17		2.601844		
	Replicates		1		.172088		
	Treatments		8		1.953044		
R^2 = .979	Due to regression	5		1.911847		.382369	6.42**
R = .989	Lack of fit	3		.041197			
	Error		8		.476712	.059589	

**P ≑ .01

lack of fit term should be of the same order of magnitude (or less) than the experimental error. In the cases under consideration, the lack of fit terms are considerably less than the error terms.

On the basis of these tests it is assumed that the quadratic function characterizes the data adequately.

Whether or not individual terms should be dropped from the equations can be solved statistically by computing the t values and the standard errors of each coefficient. Table 3 includes these values for the equations. If terms are to be dropped on the basis of the t test alone and assuming that the 5 per cent level of significance is the critical one, then a number of terms would be discarded. These are P^2, K^2 and PK from 4.1 and 4.4 and P^2 and PK from 4.5.

A more lenient criterion is proposed by Anderson (1). A variable is dropped only if the standard error of the regression coefficient exceeds the estimated coefficient. As far as equations 4.1, 4.4 and 4.5 are concerned this means that only the PK term would be dropped in each case.

In practice other considerations affect the final form of the equation. If the P^2 and K^2 terms are dropped from equation 4.1 this assumes linear response of alfalfa growth to all levels of P_2O_5 and K_2O fertilization. Inclusion of a squared term is not easy to interpret individually where plant growth is concerned. But as far as the whole equation

Table 3. Standard errors and t values for equations 4.1 to 4.5

	P	K	P^2	K^2	PK
Equation 4.1	.007042	.006181	-.000028	-.000027	-.000010
Standard errors	.002865	.002723	.000020	.000019	.000015
t values	2.59	2.27	1.36	1.29	.67
Probability levels of t's[a]	.01	.05	.20	.20	.50
Equation 4.2	.001278	.001403	-.000004	-.000006	.000005
Standard errors	.002028	.002033	.000015	.000015	.000011
t values	.63	.69	.26	.39	.45
Probability levels of t's[a]	.50	.50	.50	.50	.50
Equation 4.3	.001431	.002181	-.000005	-.000012	-.000002
Standard errors	.001044	.001048	.000008	.000008	.000005
t values	1.87	2.08	.58	1.46	.37
Probability levels of t's[a]	.20	.05	.50	.20	.50
Equation 4.4	.008320	.007584	-.000032	-.000033	-.000005
Standard errors	.002432	.002430	.000018	.000019	.000013
t values	3.42	3.12	1.73	1.77	.37
Probability levels of t's[a]	.01	.01	.10	.10	.50
Equation 4.5	.009751	.009765	-.000037	-.000065	-.000007
Standard errors	.003038	.003033	.000023	.000023	.000016
t values	3.21	3.22	1.59	1.92	.43
Probability levels of t's[a]	.01	.01	.15	.05	.50

[a]Probability of drawing a t value as large or larger given the null hypothesis.

goes, it does permit diminishing returns. Thus the equation gives a picture of plant growth more in keeping with accepted theory. Hence squared terms have been retained.

The inclusion of a PK term is easier to justify on biological grounds. Some interaction between P_2O_5 and K_2O might be expected which would have an influence on plant growth. Thus in spite of the fact that the t's and the standard errors for all the PK's are low, it was decided to retain this term in the equations.

D. Nature of the Production Surfaces

Regression equations 4.1, 4.4 and 4.5 were used to derive expected yields of alfalfa for various P_2O_5 and K_2O levels. These are shown in Table 4. The data from this table has been used to construct the production surfaces of Figures 1, 2, 3 and 4.

Figure 1 shows the nature of the production surface obtained when yields from the first cutting of alfalfa are considered alone. The relevant range of fertilization for the experiment was from 0-120 pounds per acre, for both nutrients. But it was felt that for illustrative purposes extrapolation beyond this range was justified. This was because of the goodness of fit of the regressions and because of the symmetrical nature of the estimating equations.

Figure 1 indicates that with K_2O held constant at some

Table 4. Expected yields of alfalfa (tons oven dry material per acre) for various P_2O_5 and K_2O levels (lbs. per acre)

	P	K 0	40	80	120	160	200
1st cut	0	.822	1.026	1.143	1.175	1.120	.978
	40	1.059	1.267	1.348	1.364	1.293	1.135
	80	1.206	1.378	1.463	1.463	1.376	1.202
	120	1.264	1.620	1.489	1.473	1.370	1.180
	160	1.232	1.372	1.425	1.393	1.274	1.068
	200	1.110	1.234	1.271	1.223	1.088	.866
1st + 2nd cut	0	1.634	1.884	2.030	2.069	2.002	1.831
	40	1.916	2.158	2.296	2.327	2.252	2.073
	80	2.095	2.329	2.459	2.482	2.399	2.212
	120	2.171	2.397	2.519	2.534	2.443	2.248
	160	2.146	2.364	2.478	2.485	2.386	2.183
	200	2.018	2.228	2.334	2.333	2.226	2.015
1st + 2nd + 3rd cut	0	2.124	2.443	2.617	2.048	2.534	2.277
	40	2.655	2.763	2.926	2.945	2.820	2.552
	80	2.667	2.964	3.115	3.124	2.987	2.708
	120	2.761	3.046	3.187	3.184	3.037	2.746
	160	2.737	3.011	3.140	3.127	2.968	2.666
	200	2.594	2.857	2.975	2.950	2.780	2.467

level, as the amount of P_2O_5 fertilizer applied per acre grows heavier the yield of alfalfa increases. The maximum yield is obtained at a P_2O_5 level of 120 pounds. Thereafter addition of further quantities of P_2O_5 cannot check the decrease in total yield. On the other hand, for low levels of P_2O_5, the maximum yield of alfalfa is obtained when the amount of K_2O required decreases. Thus when P_2O_5 is applied at the rate of 200 pounds per acre, only 80 pounds of K_2O

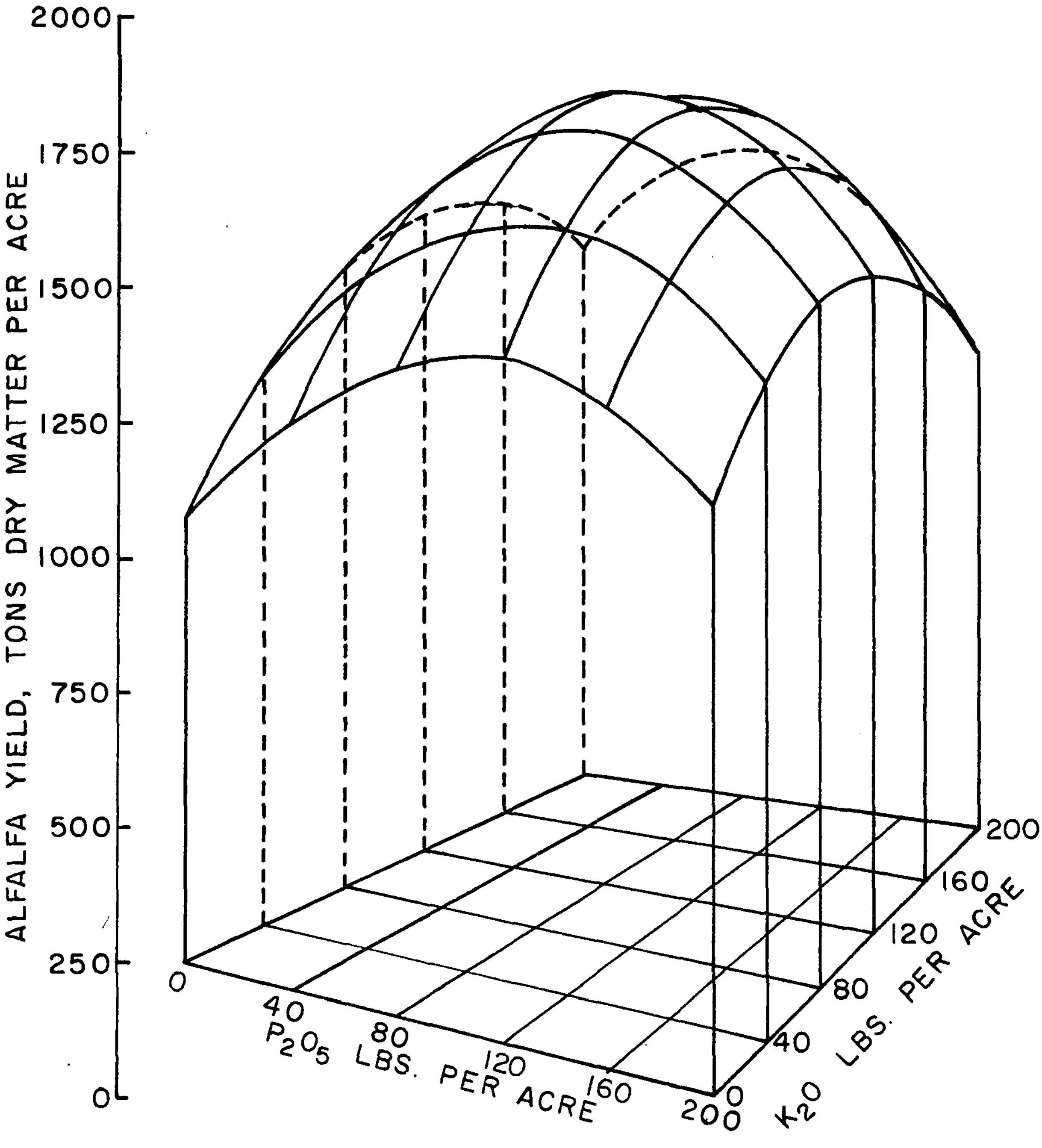

Figure 1. Production surface for 1st cutting of alfalfa

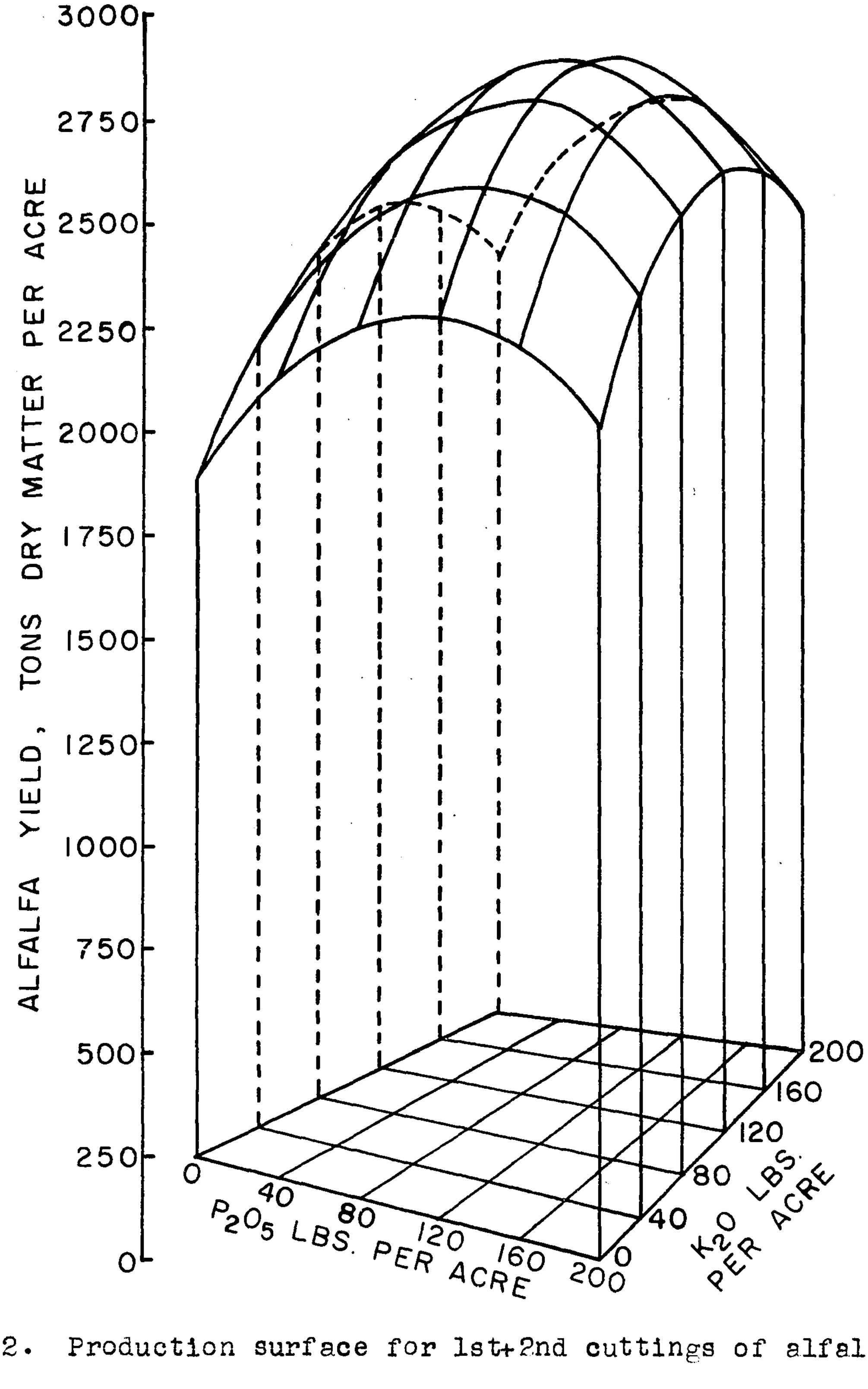

Figure 2. Production surface for 1st+2nd cuttings of alfalfa

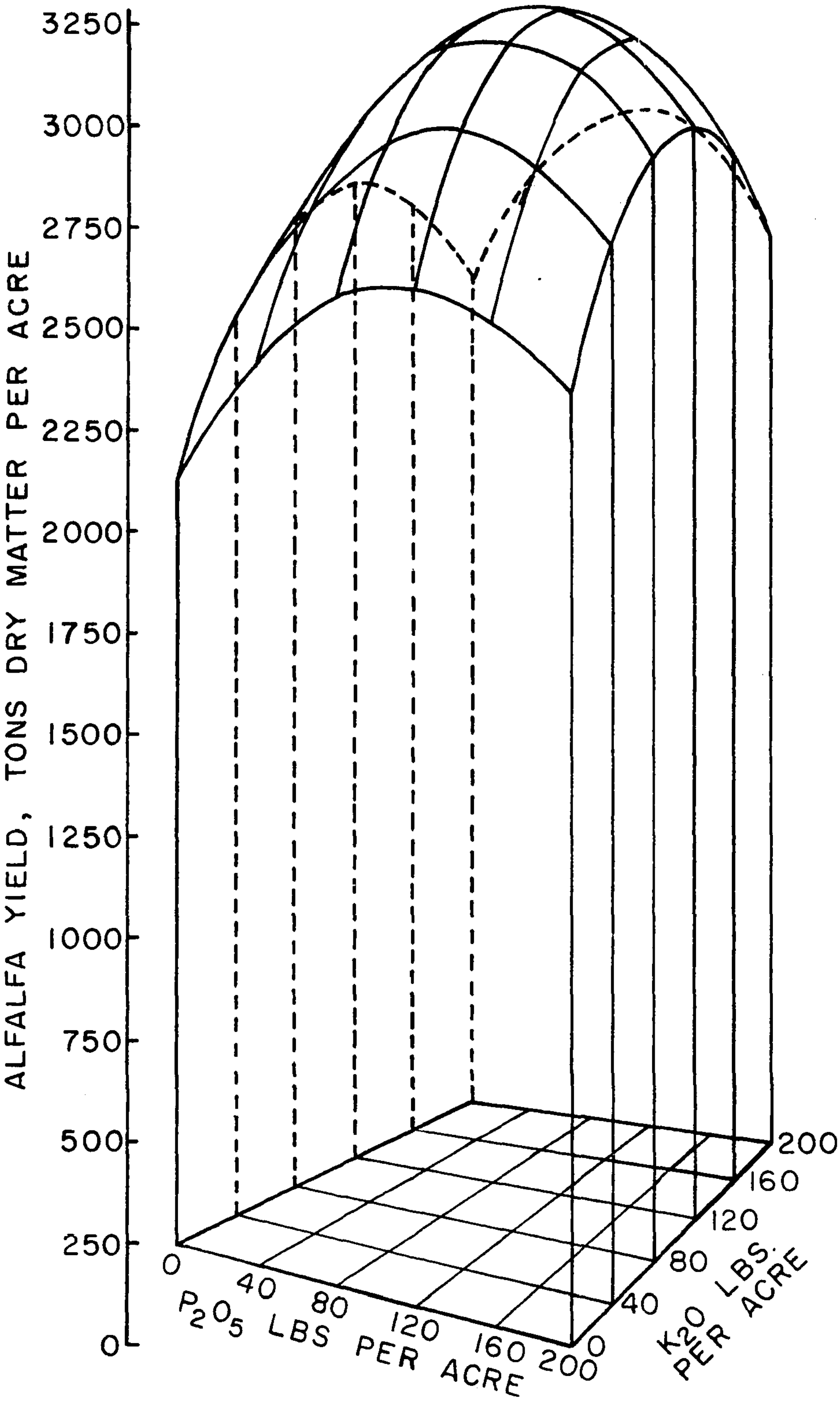

Figure 3. Production surface for 1st+2nd+3rd cuttings of alfalfa

are necessary to maximize yield.

Figure 2 illustrates the shape of the surface obtained when alfalfa yields from the first and second cuttings are aggregated. As the level of P_2O_5 is increased, while holding K_2O constant, there is a sharp yield response. Yield reaches its maximum when the quantity of P_2O_5 is approximately 120 pounds per acre (irrespective of the K_2O level). Contrariwise, when P_2O_5 remains constant, and the amount of K_2O is increased, the highest alfalfa yields occur initially when K_2O is at 120 pounds per acre. But as the P_2O_5 level increases, the amount of K_2O necessary for attainment of a maximum yield decreases to 80 pounds. Thus as in Figure 1 (under similar circumstances) the relationship is one of nutrient substitutability.

The production surface illustrated in Figure 3 is the one corresponding to the total alfalfa yield for the whole season (first plus second plus third cuttings). When K_2O is held constant and P_2O_5 is increased, then, as in the two previous cases, the maximum alfalfa yield is obtained when P_2O_5 is applied at 120 pounds per acre. But P_2O_5 may be held at various levels and K_2O increased. Then for all levels of P_2O_5 an application of 100 pounds per acre of K_2O gives a maximum alfalfa yield.

Figure 4 brings Figures 1, 2 and 3 together for compara-

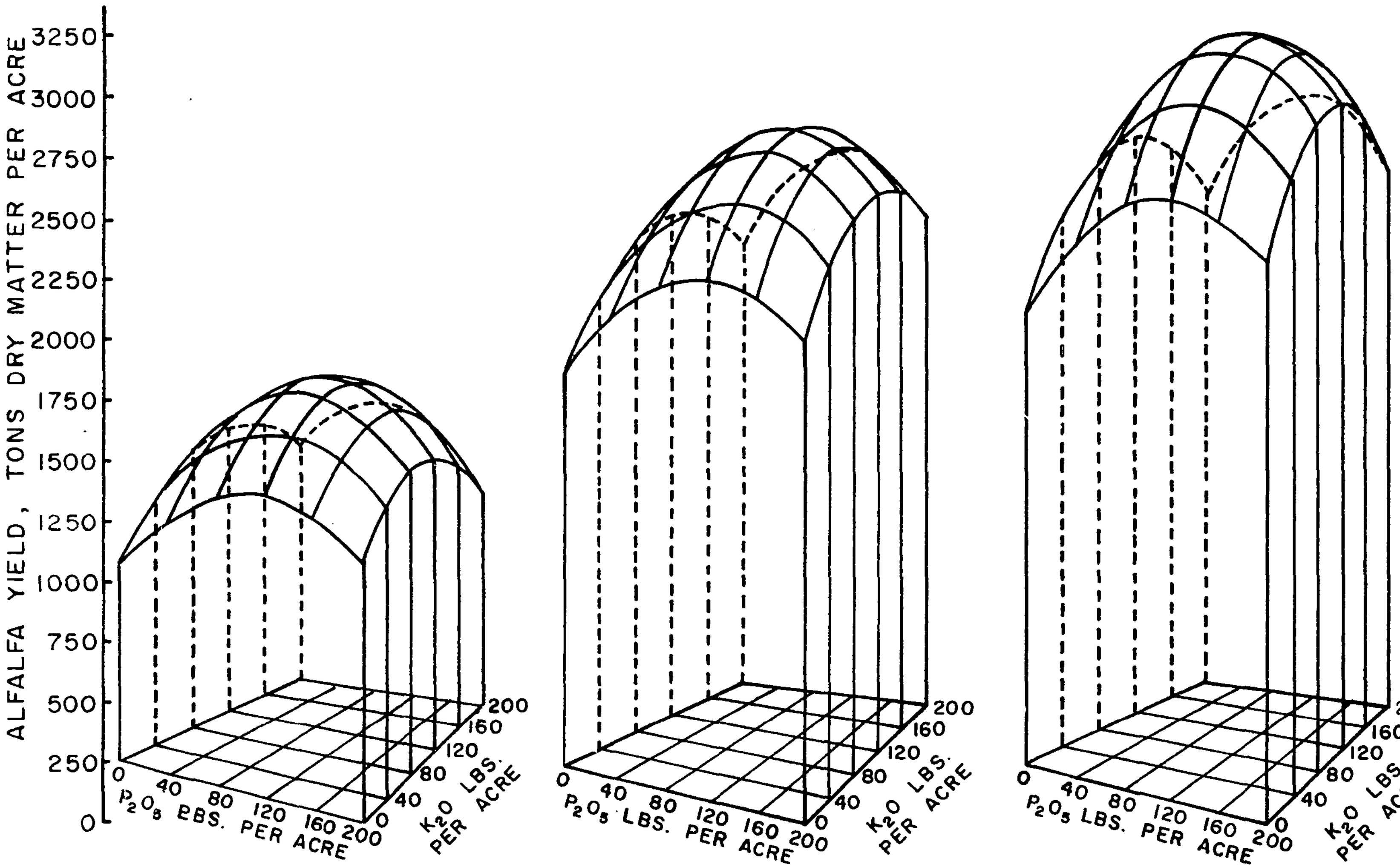

Figure 4. Production surfaces for 1st, 1st+2nd and 1st+2nd+3rd cuttings of alfalfa

tive purposes. A noticeable feature of the diagram is that the three surfaces are all good examples of the classical increasing-decreasing total returns pattern. The comparative heights of each surface are a reflection of the total yield after each cutting. The differences in heights represent the addition to total yield due to the extra cutting. The second cutting was heavier than the third (as is reflected in equations 4.2 and 4.3). Hence the increase in height of the second surface from the first is greater than the increase from the second to the third.

E. Nature of the Yield Isoquants

Isoquants were derived from the basic production functions 4.1, 4.4 and 4.5 by assuming values for one nutrient and yield (Y) and solving for the other nutrient. Isoquant equations for the first cutting (4.6), the first plus second cuttings (4.7) and the first plus second plus third cuttings (4.8) are shown on page 25.

Equations 4.6, 4.7 and 4.8 were used to derive the isoquants shown in Figure 5. A set of three isoquants are shown for each equation. The isoquants predict various combinations of P_2O_5 and K_2O required to produce a particular alfalfa yield. Some of these combinations are shown in Table 5. Thus for one cutting, 5 pounds of K_2O and 35 pounds of P_2O_5 <u>or</u> 25 pounds of K_2O and 7 pounds of P_2O_5 give a yield of 1.2

$$P = \frac{(.007887-.000011K) \pm \sqrt{(.007887-.000011K)^2-4(.000031)(Y-.006923K+}}{.000062}$$

$$P = \frac{(.009318-.000006K) \pm \sqrt{(.009318-.000006K)^2-4(.000036)(Y-.008494K}}{.000072}$$

$$P = \frac{(.010921-.000008K) \pm \sqrt{(.010921-.000008K)^2-4(.000041)(Y-.010937K}}{.000082}$$

$$\ldots 23K+.000030K^2-.920903 \qquad (4.6)$$

$$\ldots 3494K+.000037K^2-1.830284 \qquad (4.7)$$

$$\ldots 0937K+.000050K^2-2.379394 \qquad (4.8)$$

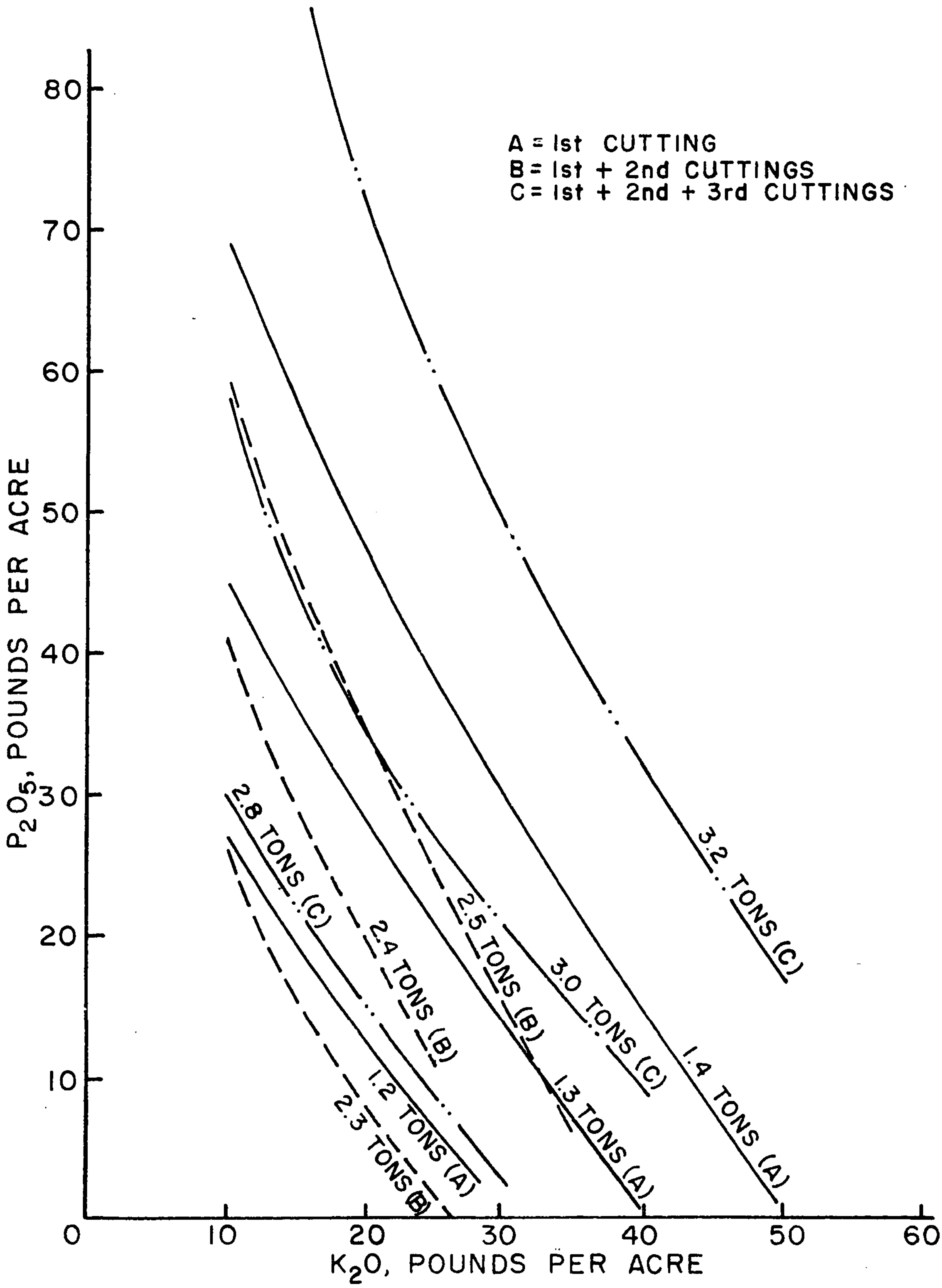

Figure 5. Isoquants for 1st, 1st+2nd and 1st+2nd+3rd cuttings of alfalfa hay

Table 5. Fertilizer combinations and corresponding marginal rates of substitution for various hay yields

Lb. K_2O	Lb. P_2O_5	MRS[a] P_2O_5 for K_2O	K_2O	Lb. P_2O_5	MRS[a] P_2O_5 for K_2O	K_2O	Lb. P_2O_5	MRS[a] P_2O_5 for K_2O
1st cut 1.2 tons per acre			1st cut 1.3 tons per acre			1st cut 1.4 tons per acre		
5	35	.908	5	54	.744	5	84	.460
10	27	1.013	10	45	.856	10	69	.629
20	13	1.230	20	28	1.095	20	49	.893
25	7	1.343	30	14	1.346	30	31	1.178
30	0	1.475	40	0	1.656	50	2	1.849
1st+2nd cut 2.2 tons per acre			1st+2nd cut 2.3 tons per acre			1st+2nd cut 2.4 tons per acre		
5	37	.838	5	53	.701	5	75	.507
10	26	.972	10	41	.840	10	59	.677
15	16	1.108	15	30	.981	20	37	.962
20	8	1.238	20	20	1.125	30	15	1.303
25	0	1.380	25	11	1.273	35	6	1.479
1st+2nd+3rd cut 2.8 tons per acre			1st+2nd+3rd cut 3.0 tons per acre			1st+2nd+3rd cut 3.2 tons per acre		
5	38	.766	5	69	.528	10	108	.219
10	30	.864	10	59	.634	20	79	.516
15	22	.971	20	40	.868	30	49	.883
20	15	1.079	30	23	1.134	40	30	1.216
30	2	1.328	40	9	1.437	50	17	1.573

[a]Pounds of K_2O replaced by 1 pound of P_2O_5.

tons of hay.

The isoquants are curved just enough to indicate diminishing marginal rates of substitution. The change in slope from left to right is gradual, indicating that the nutrients are good substitutes, within the range of the experiment. This is true for each set of isoquants.

Table 5 also includes marginal rates of substitution of P_2O_5 for K_2O. The marginal rate of substitution may be defined as the ratio of the marginal products of the two inputs. The rates of substitution were derived from the left hand sides of equations 4.9, 4.10 and 4.11 by substituting values of K_2O and P_2O_5 into the equations. The marginal rate of substitution indicates the change in the amount of one input necessary to maintain a certain yield, when one unit of another input is added. In Table 5 the isoquant equation for the 3.2 ton yield of the first plus second plus third cuttings slopes sharply at its upper end. To maintain yield 1 pound of P_2O_5 replaces a small amount of K_2O. However as the isoquant flattens out the amount of K_2O replaced by each pound of P_2O_5 becomes greater.

F. Nature of Yield Isoclines

Yield isoclines (least cost expansion paths) were derived by equating the marginal products of each production function to the nutrient price ratio and solving for one nutrient. Isoclines were worked out for the first, first plus second, and first plus second plus third cuttings, so that the relevant production functions were 4.1, 4.4 and 4.5. The isocline equations corresponding to these functions are presented below in the order mentioned. The nutrient price ratio is represented by "a".

$$\frac{.007887 - .000062P - .000011K}{.006923 - .000060K - .000011P} = a \qquad (4.9)$$

$$\frac{.009318 - .000072P - .000006K}{.008494 - .000074K - .000006P} = a \qquad (4.10)$$

$$\frac{.010921 - .000082P - .000008K}{.010937 - .000100K - .000008P} = a \qquad (4.11)$$

In Figure 6 an isocline family has been drawn for each equation. Each isocline represents the least cost P_2O_5 and K_2O combination for the nutrient price ratio shown. (The actual P_2O_5 and K_2O prices are 10 cents and 5 cents per pound, respectively, so that in reality pP/pK is 2.0.) The relative slopes of each set of isoclines do not differ greatly. As an isocline is a line connecting all points of equal slope on a family of isoquants it could be inferred that the slopes of the corresponding sets of isoquants are also similar. This is confirmed in Figure 5. The dotted lines are the ridge lines (i.e., isoclines representing zero substitution rates) beyond which the inputs will not substitute for each other.

On any production surface where the yield attains a maximum, the family of isoclines converge to a point. The point is where the partial derivatives of both inputs are zero. Maximum yields for the cuttings are predicted at similar fertilization rates. For one cutting alone, the amounts

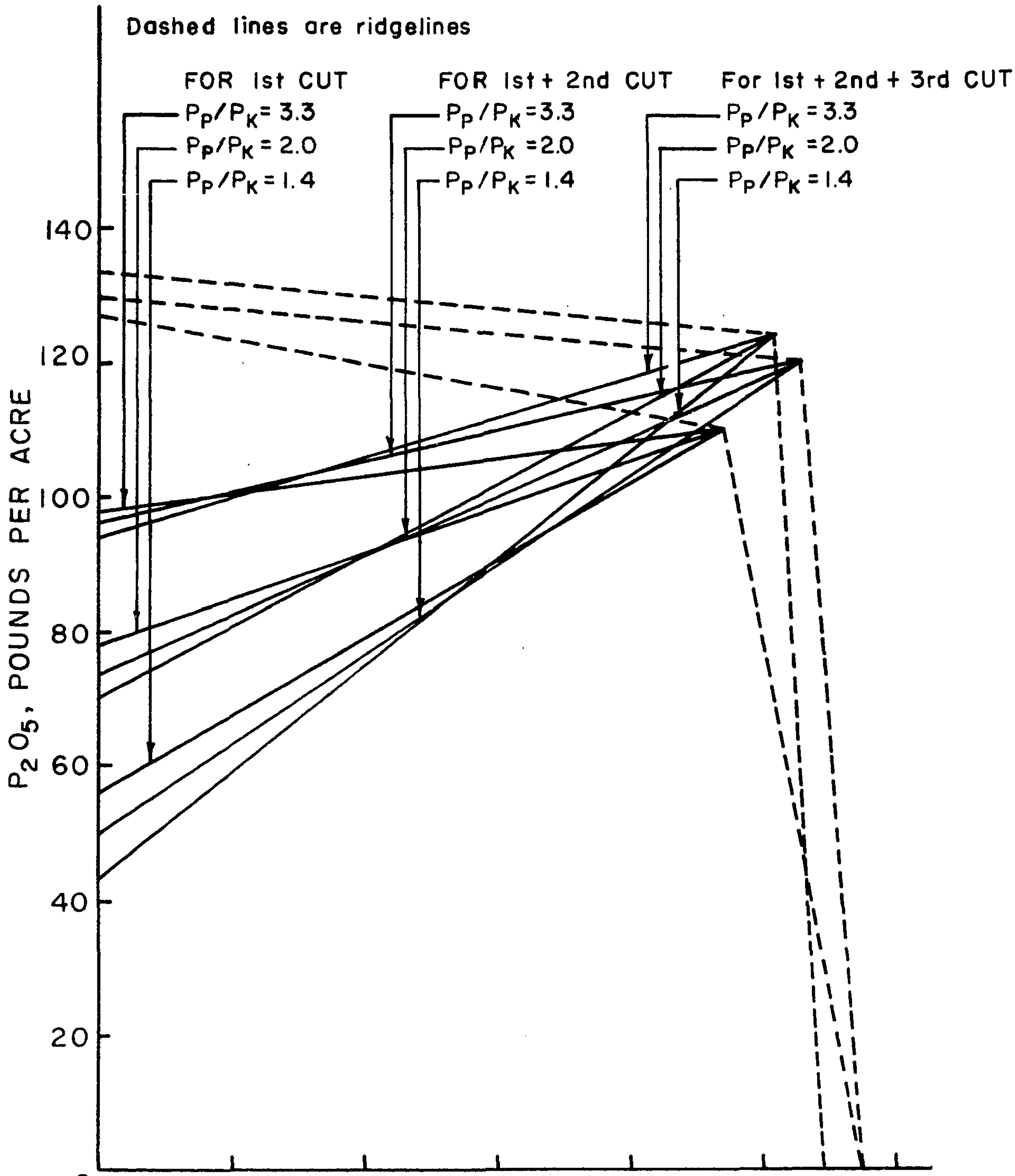

Figure 6. Isoclines for 1st cut, 1st+2nd cut and 1st+2nd+3rd cuts of alfalfa hay

of P_2O_5 and K_2O required are smaller than for two or three cuttings. To maximize yield for three cuttings more P_2O_5 but less K_2O is required than for two cuttings.

The first two objectives of this study have now been attained. A function has been chosen and fitted. Statistical tests have shown that it characterizes the data adequately. In addition, production surfaces, isoquants and isoclines have been derived which conform to production function theory. Chapter V will make use of the regression equations to derive profit maximizing quantities of fertilizer.

V. DERIVATION OF ECONOMIC OPTIMA

A. Introduction

Due to past experience, there may be little uncertainty in the farmers' mind as to the number of cuttings of alfalfa expected. It is possible that he is not only subjectively certain, but correct in his estimates. In any case he discriminates in use of fertilizer applied to alfalfa in the early spring. The present chapter examines the profit maximizing implications of vario us rates of fertilization. The basic assumption is that the farmer has <u>ex ante</u> knowledge of the number of cuttings expected.

Throughout the analysis it is necessary that a money value be put on the hay crop, so that various optima may be worked out. This value is taken as the local market sale price. The justification for this approach is that should a farmer wish to sell hay, its value is no greater than the current market price. On the other hand he cannot impute a higher than market value to his own hay. If he found that his cost of production was higher than average, then, other things being equal, he should buy, rather than make, hay. Appendix A shows the range in prices received by Iowa farmers for alfalfa hay for the period 1946-1958. Hay prices used in subsequent computations are based on this range.

The discussion in this chapter is confined to four possibilities. The first and simplest case is the derivation of profit maximizing quantities of fertilizer under the assumption that the farmer has unlimited capital available for its purchase. The second case is concerned with maximization of returns per dollar invested in fertilizer. The third possibility is that common fertilizer mixes are used rather than specific quantities. The effect on profits of using mixtures is shown. The fourth case centers around the differences in net returns per dollar invested in fertilizer, if mixtures are used. Finally graphs are presented to illustrate the equating of marginal returns when fertilizer is used for different crops. In working out profits due to fertilization the only cost taken into account is that of the fertilizer. If harvesting costs are included profits would be lowered. However as the analysis is primarily concerned with methodology, fertilizer costs are assumed representative of all costs. Similarly hay prices in the analysis are presumed inclusive of all costs such as transportation and dealer charges.

B. The Non-competitive Resource, Unlimited Capital Situation

Under this particular situation it is assumed that the farmer is concerned only with maximizing net returns. At this juncture the analysis is not concerned with evaluating returns from employment of capital in different resource-product situations. There is no limitation on the amount of money available for fertilizer purchase.

Profit maximizing quantities of fertilizer are obtained by taking the derivatives of the original production function equations (4.1, 4.4 and 4.5) with respect to P and K. The derivatives of each function (equated to the nutrient / hay price ratio) are shown below; 5.1 and 5.2 correspond to the first alfalfa cutting, 5.3 and 5.4 to the first plus second cuttings and 5.5 and 5.6 to the first plus second plus third cuttings.

$$.007887 - .000062P - .000011K = pP/pH \qquad (5.1)$$

$$.006923 - .000060K - .000011P = pK/pH \qquad (5.2)$$

$$.009318 - .000072P - .000006K = pP/pH \qquad (5.3)$$

$$.008494 - .000074P - .000006P = pK/pH \qquad (5.4)$$

$$.010921 - .000082P - .000008K = pP/pH \qquad (5.5)$$

$$.010937 - .000100K - .000008P = pK/pH \qquad (5.6)$$

Various hay and fertilizer prices are assumed and each set (5.1 and 5.2; 5.3 and 5.4; 5.5 and 5.6) is solved simultaneously to give profit maximizing quantities of fertilizer nutrients. pP and pK represent the prices of P_2O_5 and K_2O fertilizer per pound, and pH is the market price for hay. The profit maximizing rates of fertilization are presented in Tables 6, 7 and 8.

The tables confirm elementary input-output theory. As

Table 6. 1st cut alfalfa - profit maximizing rates of P and K fertilization for various hay and fertilizer prices

Price hay ($/ton)	Price fertilizer (cents/lb.)		Profit maximizing quantities of fertilizer		Hay yield (tons/acre)
	P_2O_5	K_2O	P_2O_5	K_2O	
15	8	3	27	77	1.443
15	10	5	9	58	1.284
15	12	7	0	27	1.085
20	8	3	48	82	1.551
20	10	5	35	67	1.462
20	12	7	21	53	1.343
25	8	3	61	84	1.600
25	10	5	50	73	1.543
25	12	7	39	62	1.468
30	8	3	69	86	1.626
30	10	5	60	77	1.587
30	12	7	51	67	1.534

the price of hay increases it pays to apply more fertilizer. But as fertilizer prices increase (with the price of hay remaining constant) net profits are maximized by restricting fertilizer use. Thus in Table 6 if the hay price rises from \$15 to \$20 per ton, and the price of P_2O_5 remains at 8 cents per pound, the amount of P_2O_5 fertilizer which maximizes profits increases from 27 to 48 pounds per acre. If the hay price doubles (from \$15 to \$30) the profit maximizing quantity of fertilizer rises still further from 27 to 69 pounds. On the other hand if hay remains at \$20, profits are maximized

Table 7. 1st+2nd cut alfalfa - profit maximizing rates of P and K fertilization for various hay and fertilizer prices

Price hay ($/ton)	Price fertilizer (cents/lb.) P_2O_5	Price fertilizer (cents/lb.) K_2O	Profit maximizing quantities of fertilizer P_2O_5	Profit maximizing quantities of fertilizer K_2O	Hay yield (tons/acre)
10	8	3	12	73	2.354
10	10	5	0	47	2.148
10	12	7	0	20	1.985
15	8	3	48	84	2.623
15	10	5	31	67	2.475
15	12	7	14	51	2.286
20	8	3	66	89	2.716
20	10	5	54	77	2.628
20	12	7	41	64	2.528
25	8	3	77	92	2.760
25	10	5	67	82	2.708
25	12	7	57	72	2.639
30	8	3	84	94	2.783
30	10	5	76	86	2.748
30	12	7	67	78	2.699

by applying more fertilizer when its cost is low but less when its cost is high.

As the number of cuttings expected increases, so does the profit maximizing quantity of fertilizer. Assume that hay is selling for $20 per ton and the price of P_2O_5 and K_2O is 10 and 5 cents per pound, respectively. For one cutting, application of 35 pounds of P_2O_5 and 67 pounds of K_2O maximizes profits. For two cuttings the profit maximizing

Table 8. 1st+2nd+3rd cut alfalfa - profit maximizing rates of P and K fertilization for various hay and fertilizer prices

Price hay ($/ton)	Price fertilizer (cents/lb.) P_2O_5	K_2O	Profit maximizing quantities of fertilizer P_2O_5	K_2O	Hay yield (tons/acre)
10	8	3	28	77	3.181
10	10	5	5	59	2.901
10	12	7	0	39	2.729
15	8	3	60	85	3.414
15	10	5	45	72	3.290
15	12	7	30	60	3.131
20	8	3	76	88	3.494
20	10	5	64	79	3.422
20	12	7	53	70	3.334
25	8	3	85	91	3.530
25	10	5	76	83	3.485
25	12	7	67	76	3.429
30	8	3	92	92	3.552
30	10	5	84	86	3.520
30	12	7	77	80	3.483

quantities rise to 54 pounds of P_2O_5 and 77 pounds of K_2O, while for three cuttings 64 pounds and 79 pounds, respectively, are required.

As hay prices rise the proportions of nutrients which maximize profits change considerably. In Table 8 assume that P_2O_5 costs 8 cents per pound and K_2O costs 3 cents per pound. When hay is selling at $10 per ton the proportions of P_2O_5 and K_2O which maximize profits are approximately 1 : 3. But if the hay price rises to $30, the ratio changes to 1 : 1. As nutrient prices change, however, the profit maximizing proportions of fertilizer change in the same direction. In Table 7 assume a hay price of $25 per ton. The amounts of P_2O_5 and K_2O which maximize profits are in a 1 : 1.2 ratio whatever the individual nutrient prices may be.

If Tables 6, 7 and 8 are compared they substantiate a fairly evident truth. This is: as the expectation of the number of cuts per year increases, a higher level of fertilization is required for profit maximization. More importantly, the comparison emphasizes that correct anticipation of the number of cuts (and thus the <u>ex</u> <u>ante</u> decision to fertilize accordingly) can have significant consequences on costs. For example the farmer may expect two cuttings and apply fertilizer with this in mind. But only one cut is obtained. Assume a hay price of $15 per ton and that P_2O_5 and K_2O cost 8 and 3 cents per pound, respectively. Tables 6 and 7 show that the farmer has applied 21 pounds of P_2O_5 and 7 pounds of K_2O too much per acre. This represents $1.89 per acre wasted in excess fertilizer. If fertilizer prices rise so that P_2O_5 and K_2O now cost 12 and 7 cents per pound, respectively, the excess fertilizer is valued at $3.36 per acre. Or, the farmer may fertilize for three cuts and get only two. With hay at $15 per ton, P_2O_5 at 12 cents and K_2O at 7 cents per pound the incorrect decision costs $2.25 per acre (Tables 7 and 8). If the total alfalfa acreage is taken into account the difference is large. Further attention is given to this problem in Chapter VI.

C. The Non-competitive Resource, Limited Capital Situation

The previous case assumed that the farmer had unlimited capital available for the purchase of fertilizer. But most farmers have to ration their available capital among different inputs. Hence he may be more concerned with maximizing returns per dollar invested in fertilizer. In this situation, fertilizer should be added only to the point where the marginal value product per dollar invested is a maximum. Fertilizer is not applied to the stage where each dollar spent is just returned through increased value product. The aim is to cease fertilizer application when marginal value product is highest. The amount of fertilizer which maximizes returns per dollar invested, may be derived as follows:[1]

Consider a production function of the form

$$Y = a + bP - cP^2 .$$

Assume that Y is product output and P is fertilizer input. If e is the money value of the return per unit of product, a value function may be set up as follows

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[1]Earl O. Heady and J. T. Pesek. Ames, Iowa. Minimum fertilizer quantities. Private communication. 1958.

$$V = ea + ebP - ecP^2 .$$

A cost function, C, may also be constructed,

$$C = f + gP .$$

Here, f is the fixed cost associated with application of fertilizer per unit of area and g is the price per pound of P. The return per dollar invested in fertil izer may be expressed as

$$I = \frac{ea + ebP - ecP^2}{f + gP} .$$

The return on the money invested is maximized by taking the partial derivative of I with respect to P. P is then solved for, assuming various e, f and g values.

The above analysis may be applied to the alfalfa data of this study. The alfalfa fertilization problem involves two nutrients P_2O_5 and K_2O. If one nutrient is held constant in the basic production functions 4.1, 4.4 and 4.5, a value function may be derived for the remaining nutrient. If fixed costs and fertilizer costs are then assumed as in Table 9 the amounts of fertilizer maximizing returns per dollar invested can then be worked out. Table 9 indicates for a hay price of $20 the amounts of fertilizer to use so that the return per dollar invested is a maximum. The fixed costs are based on records kept at Iowa State College (15). They include depreciation, interest, housing, repairs, fuel and labor. The average per acre fixed cost is taken as $1.30, but high and low cost levels have also been assumed. These correspond

Table 9. Fertilizer quantities maximizing return per dollar invested

Cutting	Hay price ($/ton)	Fixed cost ($/acre)	Fertilizer prices (cents/lb.) P_2O_5	K_2O	Maximizing quantities of fertilizer (lbs.) P_2O_5	K_2O	Hay yield (tons/acre)
1st	20	.80	8	3	0	56	1.215
		1.30	10	5	0	56	1.215
		1.80	12	7	0	56	1.215
		.80	8	3	40	83	1.517
		1.30	10	5	40	79	1.511
		1.80	12	7	40	77	1.507
		.80	8	3	80	86	1.651
		1.30	10	5	80	82	1.648
		1.80	12	7	80	81	1.647
		.80	8	3	120	83	1.679
		1.30	10	5	120	81	1.678
		1.80	12	7	120	80	1.677
1st+2nd	20	.80	8	3	0	56	2.190
		1.30	10	5	0	55	2.185
		1.80	12	7	0	55	2.185
		.80	8	3	40	85	2.580
		1.30	10	5	40	81	2.571
		1.80	12	7	40	79	2.566
		.80	8	3	80	91	2.768
		1.30	10	5	80	88	2.763
		1.80	12	7	80	86	2.760
		.80	8	3	120	93	2.833
		1.30	10	5	120	90	2.829
		1.80	12	7	120	88	2.827
1st+2nd +3rd	20	.80	8	3	0	54	2.824
		1.30	10	5	0	54	2.824
		1.80	12	7	0	54	2.824
		.80	8	3	40	81	3.282
		1.30	10	5	40	77	3.271
		1.80	12	7	40	76	3.268
		.80	8	3	80	87	3.509
		1.30	10	5	80	84	3.503
		1.80	12	7	80	82	3.500
		.80	8	3	120	88	3.591
		1.30	10	5	120	86	3.588
		1.80	12	7	120	87	3.590

with the high and low fertilizer prices. As the amount of P_2O_5 applied per acre grows heavier, the amount of K_2O required does not increase in proportion. This suggests that use of large amounts of K_2O at higher P_2O_5 levels does not have a beneficial yield effect. The main conclusion to be drawn from the table is that even if one, two or three cuttings are expected, the amounts of fertilizer recommended are still quite similar. Compared with the relevant portions of Tables 6, 7 and 8, fertilization rates are not appreciably lower. Nor is there a noticeable difference in yields. Accurate comparison is difficult because in Table 9 P_2O_5 is held constant at various levels.

D. Non-competitive Resource Limited Capital Situation, Using Common Fertilizer Mixes

Throughout the preceding analysis no account has been taken of the fact that in practice a number of farmers use fertilizers in which the nutrient elements are already mixed in some set proportion. This may be due to personal preference, district practice or conservatism. Whatever the cause, the results are that yields may not be as favorable and fertilizer costs are also increased.

A recent Iowa survey[1] found that the three most commonly

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[1]John Harp. Ames, Iowa. Fertilizer dealer study. Private communication. 1958.

used PK mixtures are 0-20-20, 0-35-15 and 0-12-36. Expected alfalfa yields using 50 pound increments of these fertilizers were computed from the basic production functions 4.1, 4.4 and 4.5. Tables 10, 11 and 12 indicate the net returns when the alfalfa is sold as hay. The fertilizer mixtures are valued at current market prices.

Table 10 deals with the first cut. If the price of hay is $15 per ton, the greatest net return is obtained by applying 150 pounds per acre of the 0-12-36 mixture. This gives a net return (neglecting fixed and other costs) of $15.42 per acre. By comparison, 150 pounds of 0-20-20 gives a net return of $15.00 per acre, while 100 pounds per acre of 0-35-15 returns $14.50. As the price of hay rises so does the profit maximizing quantity of fertilizer. For a hay price of $25 per ton, maximum net returns are obtained using 300 pounds of 0-20-20. The net value product here is $29.77 per acre. When 250 pounds per acre of 0-12-36 is used, net returns are $29.50, compared with $28.47 using 200 pounds per acre of 0-35-15. When hay is selling at $15 per ton, net returns from a per acre dressing of more than 300 pounds of 0-20-20 or 0-12-36, or 150 pounds of 0-35-15, do not pay for the cost of the fertilizer.

Table 11 outlines the returns expected when two cuts of hay are harvested. With a hay price of $20 per ton the farmer

Table 10. 1st cut alfalfa - net returns for various fertilizer quantities and hay prices

Type fertilizer	Amount applied (lbs./acre)	Fertilizer cost ($/acre)	Yield hay (tons)	Return less fertilizer cost when hay price per ton is $15	$20	$25
0-20-20	0	0	.921	13.81	18.42	23.02
	50	1.50	1.067	14.50	19.84	25.17
	100	3.00	1.188	14.82	20.76	26.70
	150	4.50	1.300	15.00	21.50	28.00
	200	6.00	1.398	14.97	21.96	28.95
	250	7.50	1.481	14.71	22.42	29.52
	300	9.00	1.551	14.26	22.02	29.77
	350	10.50	1.605	13.57	21.60	29.62
	400	12.00	1.645	12.67	20.90	29.12
	450	13.50	1.670	11.55	19.90	28.25
	500	15.00	1.682	10.23	18.64	27.05
	550	16.50	1.678	8.67	17.06	25.45
	600	18.00	1.661	6.92	15.22	23.52
0-35-15	0	0	.921	13.81	18.42	23.02
	50	2.12	1.098	14.35	19.84	25.33
	100	4.25	1.250	14.50	20.75	27.00
	150	6.37	1.377	14.28	21.17	28.05
	200	8.50	1.479	13.68	21.08	28.47
	250	10.62	1.555	12.70	20.48	28.25
	300	12.75	1.606	11.34	19.37	27.40
	350	14.87	1.632	9.61	17.77	25.93
	400	17.00	1.632	7.48	15.64	23.80
	450	19.12	1.608	5.00	13.04	21.08
	500	21.25	1.558	2.12	9.91	17.70
0-12-36	0	0	.921	13.81	18.42	23.02
	50	1.50	1.081	14.71	20.12	25.52
	100	3.00	1.217	15.25	21.34	27.42
	150	4.50	1.328	15.42	22.06	28.70
	200	6.00	1.416	15.20	22.32	29.40
	250	7.50	1.480	14.70	22.10	29.50
	300	9.00	1.520	13.80	21.40	29.00
	350	10.50	1.538	12.57	20.26	27.95
	400	12.00	1.527	10.90	18.54	26.17
	450	13.50	1.494	8.91	16.38	23.85
	500	15.00	1.438	6.57	13.76	20.95

Table 11. 1st+2nd cut alfalfa - net returns for various fertilizer quantities and hay prices

Type fertilizer	Amount applied (lbs./acre)	Fertilizer cost ($/acre)	Yield hay (tons)	Return less fertilizer cost when hay price per ton is $15	$20	$25
0-20-20	0	0	1.830	27.45	36.60	45.75
	50	1.50	2.000	28.50	38.50	48.50
	100	3.00	2.155	29.32	40.10	50.87
	150	4.50	2.293	29.89	41.36	52.82
	200	6.00	2.416	30.24	42.32	54.40
	250	7.50	2.523	30.34	42.96	55.57
	300	9.00	2.615	30.22	43.30	56.37
	350	10.50	2.690	29.85	43.30	56.75
	400	12.00	2.750	29.25	43.00	56.75
	450	13.50	2.795	28.42	42.40	56.37
	500	15.00	2.821	27.31	41.42	55.52
	550	16.50	2.834	26.01	40.18	54.35
	600	18.00	2.830	24.45	38.60	52.75
	650	19.50	2.811	22.66	36.72	50.78
0-35-15	0	0	1.830	27.45	36.60	45.75
	50	2.12	2.043	28.52	38.74	48.95
	100	4.25	2.228	29.17	40.31	51.45
	150	6.37	2.385	29.40	41.33	53.25
	200	8.50	2.515	29.22	41.80	54.37
	250	10.62	2.617	28.63	41.72	54.80
	300	12.75	2.690	27.60	41.05	54.50
	350	14.87	2.736	26.17	39.85	53.53
	400	17.00	2.755	24.32	38.10	51.87
	450	19.12	2.745	22.05	35.78	49.50
	500	21.25	2.708	19.37	32.91	46.45
	550	23.37	2.644	16.29	29.51	42.73
0-12-36	0	0	1.830	27.45	36.60	45.75
	50	1.50	2.025	28.87	39.00	49.12
	100	3.00	2.192	29.88	40.84	51.80
	150	4.50	2.331	30.46	42.12	53.77
	200	6.00	2.442	30.63	42.84	55.05
	250	7.50	2.526	30.39	43.02	55.65
	300	9.00	2.581	29.71	42.62	55.52
	350	10.50	2.609	28.63	41.68	54.72
	400	12.00	2.609	27.13	40.18	53.22
	450	13.50	2.580	25.20	38.10	51.00
	500	15.00	2.525	22.87	35.50	48.12
	550	16.50	2.441	20.12	32.32	44.52

Table 12. 1st+2nd+3rd cut alfalfa - net returns for various fertilizer quantities and hay prices

Type fertilizer	Amount applied (lbs./acre)	Fertilizer cost ($/acre)	Yield hay (tons)	Return less fertilizer cost when hay price per ton is $15	$20	$25
0-20-20	0	0	2.379	35.68	47.58	59.47
	50	1.50	2.588	37.32	50.26	63.20
	100	3.00	2.777	38.65	52.54	66.42
	150	4.50	2.946	39.69	54.42	69.15
	200	6.00	3.095	40.42	55.90	73.37
	250	7.50	3.225	40.87	57.00	73.12
	300	9.00	3.334	41.01	57.68	74.35
	350	10.50	3.424	40.86	57.98	75.10
	400	12.00	3.494	40.41	57.88	75.35
	450	13.50	3.545	39.67	57.40	75.12
	500	15.00	3.575	38.62	56.50	74.37
	550	16.50	3.586	37.29	55.22	73.15
	600	18.00	3.577	35.66	53.54	71.42
	650	19.50	3.499	32.98	50.48	67.98
0-35-15	0	0	2.379	35.08	47.58	59.47
	50	2.12	2.636	37.42	50.60	63.78
	100	4.25	2.860	38.65	52.95	67.25
	150	6.37	3.051	39.39	54.64	69.90
	200	8.50	3.209	39.63	55.68	71.72
	250	10.62	3.335	39.40	56.08	72.75
	300	12.75	3.427	38.65	55.79	72.92
	350	14.87	3.487	37.43	54.87	72.30
	400	17.00	3.514	35.71	53.28	70.85
	450	19.12	3.508	33.50	51.04	68.58
	500	21.25	3.469	30.78	48.13	65.47
	550	23.38	3.397	27.58	44.56	61.54
	600	25.50	3.293	23.90	40.36	56.82
0-12-36	0	0	2.379	35.68	47.58	59.47
	50	1.50	2.623	37.84	50.96	64.07
	100	3.00	2.830	39.45	53.60	67.75
	150	4.50	3.000	40.50	55.50	70.50
	200	6.00	3.132	40.98	56.64	72.30
	250	7.50	3.228	40.92	57.06	73.20
	300	9.00	3.286	40.29	56.72	73.15
	350	10.50	3.308	39.12	55.66	72.20
	400	12.00	3.292	37.38	53.84	70.30
	450	13.50	3.239	35.08	51.28	67.47
	500	15.00	3.149	32.23	47.98	63.72
	550	16.50	3.023	28.84	43.96	59.08
	600	18.00	2.859	24.88	39.18	53.48

should use 300 pounds of 0-20-20 per acre which returns $43.30. For this situation he is $1.50 per acre better off than if he had used 250 pounds per acre of 0-35-15. However using 250 pounds per acre of 0-12-36 he is only 28 cents per acre worse off than in the first case. If the hay price reaches $25 per ton and 0-20-20 fertilizer is used, 350 pounds per acre gives net returns of $56.75. If no fertilizer is applied at all, the cash value of an acre of hay is $45.75. Thus the potential increase in net returns due to using fertilizer is $11 per acre. Even if 0-35-15 fertilizer is used increased net returns (as against no fertilizer) are $9.05 per acre. If the 0-12-36 mixture is used it is possible to be $9.90 per acre better off.

Table 12 includes returns when three cuts of hay are expected. Fertilizer is applied with this in mind, and the expectations are realized. For a hay price of $25 per ton 400 pounds per acre of 0-20-20 gives the highest net returns. But if 300 pounds of 0-35-15 is used, the net value product decreases by $2.43 per acre. Use of 300 pounds per acre of 0-12-36 gives a return which is $2.20 per acre less favorable than if 400 pounds of 0-20-20 is used.

In terms of value product due to fertilizing at all, 400 pounds of the 0-20-20 mixture result in an extra return worth $15.88 per acre. This compares favorably with $13.45 per acre obtained by using the optimum amount of 0-35-15, or

$14.73 per acre using 0-12-36. These differences due to using alternative fertilizer mixtures may not appear to be very great. More significance is assumed when returns are placed on an alfalfa acreage per farm basis. Fixed costs of fertilizer application may average $1.30 per acre (15). So it is evident if the most suitable mixture is chosen the increased net return will cover fixed costs.

Consider again the data in Table 12. Even if hay is priced conservatively at $15 per ton the difference in value product due to using 300 pounds of 0-20-20, rather than 200 pounds of 0-35-15 is $1.38 per acre. When the hay price rises to $20 per acre the amount is $1.90. Similarly when hay is selling at $25 per ton the figure is $2.43 per acre. This compares the "best" and "worst" situations. If the second most suitable mixture is used, the potential difference is still $1.24, $.92 or $2.15 (depending on the hay price).

There is another aspect of fertilizer mixture use. Do significant differences in profit arise when common mixtures are used compared with use of a special mixture? The special mixture is made up for the farmer in quantities specified by soil test, production function or other data (subsequently this is referred to as an "optimum" mixture). Alfalfa yields were computed from the basic production function (equation 4.5) using "optimum" and common mixture amounts of fertilizer. Following this net profits were estimated. Table 13 indicates

Table 13. 1st+2nd+3rd cut alfalfa - net profits using various P and K fertilizer combinations

Mixture	Price hay ($/ton)	Price fertilizer ($/ton)	Amount fertilizer applied (lbs./acre) P_2O_5	K_2O	Net value product ($/acre)	Net profit over no fertilization ($/acre)
"Optimum"	15	--	45	72	41.25	5.57
"Optimum"	20	--	64	79	58.09	11.39
"Optimum"	25	--	76	83	75.38	15.91
0-20-20	15	60	60	60	41.01	5.33
0-20-20	20	60	70	70	57.98	10.40
0-20-20	25	60	80	80	75.35	15.88
0-35-15	15	85	70	30	39.63	3.95
0-35-15	20	85	87.5	37.5	56.08	8.50
0-35-15	25	85	105	45	72.92	13.45
0-12-36	15	60	24	72	40.98	5.30
0-12-36	20	60	30	90	57.06	9.48
0-12-36	25	60	30	90	73.20	13.73

the results. Data are based on the first plus second plus third cuttings.

For the alfalfa experimental data, use of either "optimum" or 0-20-20 fertilizer mixtures result in approximately the same net value product. The greatest difference occurs when hay is selling at $20 per ton. In this situation application of 64 pounds of P_2O_5 and 79 pounds of K_2O returns $.99 per acre more than use of 70 pounds of the 0-20-20 mixture. If 0-35-15 or 0-12-36 mixtures are used rather than "optimum" amounts, net profits are reduced by as much as

$2.89 or $1.91 per acre, respectively. It appears therefore that indiscriminate use of fertilizer mixtures may result in a considerable reduction in profits. On the other hand "optimum" quantities can sometimes be approximated by using one of the common mixes. In the latter case the difference in net profit may be unimportant.

E. The Competitive Resource, Limited Capital Situation

There is one other relevant set of circumstances relating to fertilizer use. Limited capital may be available for resouce inputs. The farmer may decide to use fertilizer only to the extent that the net return for each dollar invested is greater than if the money had been spent for other inputs. Alternatively (as has been outlined in Section B) fertilizer may be applied to the point where the return per dollar invested is a maximum. Whatever course is followed, a further choice is necessary. This is whether one of the more popular mixtures or an "optimum" combination should be applied. The consequences of applying various quantities of fertilizer are examined in Table 14.

Using the basic production function 4.5 (for the first plus second plus third cuttings) yields of hay were computed for the fertilizer combinations shown in Table 14. Valuing hay at $15, $20 and $25 per ton, the increases in net returns

Table 14. 1st+2nd+3rd cut alfalfa - net returns per dollar invested for various nutrient combinations

Fertilizer mixture	Cost ($/acre)	"Available" fertilizer inputs (lbs./acre)		Net return compared with no fertilizer when hay price is			Net[a] return per dollar invested when hay price is		
		P_2O_5	K_2O	$15	$20	$25	$15	$20	$25
"Optimum"	4.00	0	54	3.98	6.20	8.43	.99	1.55	2.11
"Optimum"	9.20	40	78	5.53	10.00	14.48	.60	1.09	1.57
"Optimum"	13.50	80	84	4.66	10.28	15.90	.34	.76	1.18
"Optimum"	17.60	120	86	1.84	7.88	13.93	.10	.45	.79
0-20-20	6.30	20	20	1.67	3.66	5.65	.38	.85	1.31
0-20-20	7.30	40	40	3.44	7.02	10.60	.47	.96	1.45
0-20-20	10.30	60	60	4.03	8.80	13.58	.39	.85	1.32
0-20-20	13.30	80	80	3.43	9.00	14.58	.26	.68	1.10
0-20-20	16.30	100	100	1.64	7.62	13.60	.10	.47	.83
0-35-15	6.30	25	10	.96	2.70	4.45	.22	.62	1.03
0-35-15	7.30	49	21	2.24	5.42	8.61	.31	.74	1.18
0-35-15	10.30	74	32	2.66	6.98	11.31	.26	.68	1.10
0-35-15	13.30	99	42	1.98	7.06	12.15	.15	.53	.93
0-35-15	16.30	123	53	.36	5.90	11.45	.02	.36	.70
0-12-36	4.30	12	36	2.47	4.72	6.98	.57	1.10	1.62
0-12-36	7.30	24	72	4.00	7.76	11.53	.55	1.06	1.58
0-12-36	10.30	36	108	3.31	7.84	12.38	.32	.76	1.20
0-12-36	13.30	48	144	.40	4.96	9.53	.03	.37	.72
0-12-36	16.30	60	180	-4.75	-.90	2.95	--	--	.18

[a]Increase in net returns due to fertilization divided by fertilizer cost.

due to using fertilizer were worked out. These increases were then expressed as a percentage of the total cost. The resulting figure was the return per dollar invested. The total cost figure includes the cost of the fertilizer and the fixed costs associated with spreading it.

The farmer may wish to know what amount of money should be spent on fertilizer so that each dollar invested is at least returned through the increase in value product. This assumes that costs, including overheads, are taken into account. Table 14 indicates that if the price of hay is $15 per ton, money invested in fertilizer will not be returned through increase in value product. Thus if an "optimum" amount of 40 pounds of P_2O_5 and 78 pounds of K_2O are applied each dollar invested returns only 99 cents. For the common mixes, even if the price of hay rises to $20 per ton, only $3 or $6 of 0-12-36 would justify the outlays involved ($3 worth of 0-12-36 returns $1.10 for each dollar invested). But if the hay price is $25 per ton then up to $12 per acre may be spent on 0-20-20 or $9 on one of the other two mixtures. However in all cases the greatest net returns can be obtained by applying "optimum" amounts of fertilizer. But the "optimum" amount in one year may not be "optimum" in another year. In an <u>ex post</u> sense common mixes will always compare unfavorably[1] with "optimum" amounts. The important question is whether the difference is significant enough to influence <u>ex ante</u> recommendations to farmers. Input-output data concerning the soil type and locality may be available for a number of years. In this case enough precision may be incorporated into management advice so that use of different mixtures results in quite dissimilar cash returns.

If capital for fertilizer is limited, interest may center around the quantity of fertilizer which maximizes returns per dollar invested. Use of 54 pounds of K_2O returns $2.11 per dollar invested when hay is selling at $25 per ton. By comparison the most profitable mixture is 100 pounds of 0-12-36 which returns $1.62. However if 66 pounds of the 0-35-15 mixture are used the returns per dollar invested fall to $1.03. These differences may be significant whn capital is limited.

The concept of maximizing returns on fertilizer investment may be expressed graphically (as in Figure 7) by plotting marginal returns curves. For an investment of $6 in fertilizer the "optimum" amount gives the greatest marginal returns. This is followed by 0-12-36, 0-20-20 and 0-35-15 in that order.

In terms of competetive inputs for the limited capital

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[1]Unless, by chance, an "optimum" quantity corresponds to some common mixture combination.

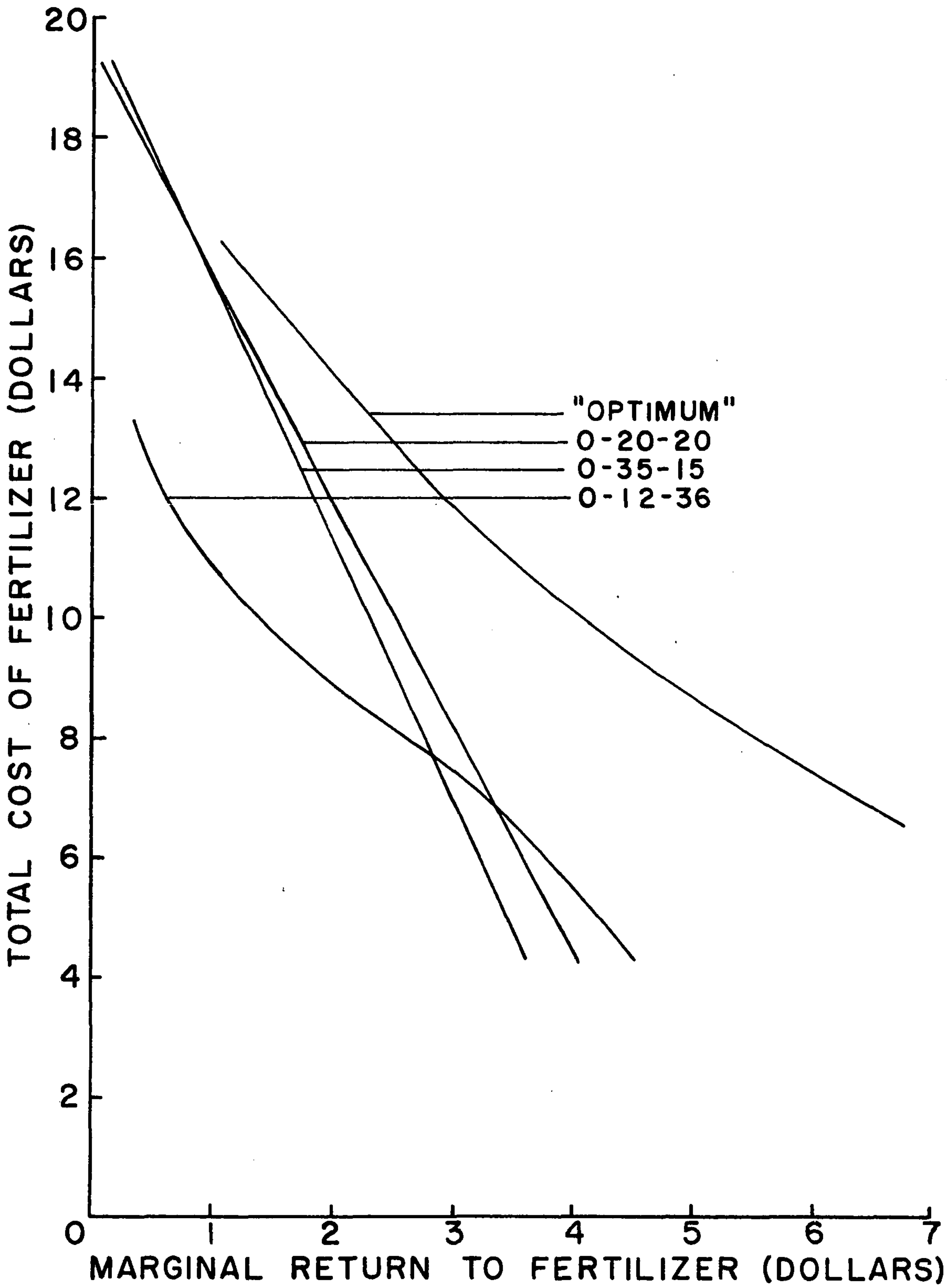

Figure 7. Marginal returns from alfalfa fertilization using common mixtures and "optimum" amounts

situation, the farmer may be more concerned with equating marginal returns for fertilizer among different crops. This is in contrast to the generally envisaged situation where marginal returns are equated between quite different farm enterprises. For example, equation of returns per extra dollar spent on feed for pigs, against labor in the cow barn.

Figure 8 illustrates a theoretical solution to the problem of equating marginal returns when fertilizer is used for different crops. Apart from the alfalfa curve, the curves are hypothetical. It is assumed that there is the same acreage in each crop. The farmer has $30 to buy fertilizer for each acre of alfalfa, corn, oats and soybeans combined. The solution is not to spend $7.50 per acre on each crop; $4 per acre is spent on fertilizer for soybeans, $7 for corn and oats and $12 on alfalfa. The marginal returns are then equated at $6 for each crop. Alternatively if $42 was available, equi-marginal returns would be secured when $8 per acre was spent on fertilizer for corn, $9 for soybeans, $11 for oats and $14 per acre for alfalfa.

This chapter has shown that the increase in net returns per acre due to fertilization can be important. The same conclusion applies whether capital is limited or unlimited. Use of different fertilizer mixtures also changes profits. The above analysis assumes that the number of cuttings is known with certainty. This assumption is discarded in the next chapter.

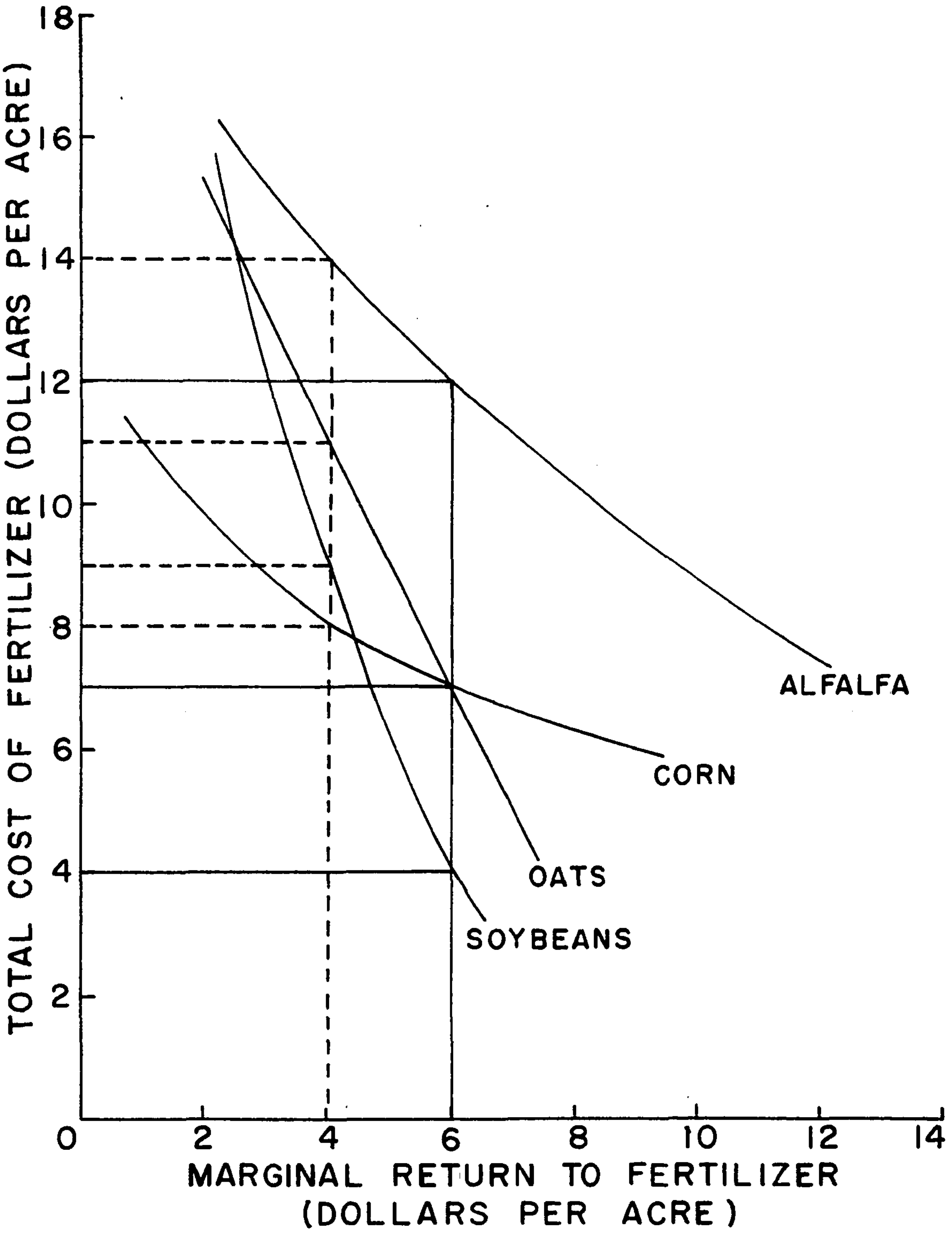

Figure 8. Marginal returns from crop fertilization (hypothetical data)

VI. NUMBER OF CUTTINGS UNCERTAIN

A. Introduction

The theme of the previous chapter was that there was no uncertainty regarding the number of alfalfa cuttings. Thus profit maximizing quantities of fertilizer could be derived. In actual fact farmers are uncertain regarding future hay yields.

This chapter proceeds a step further towards reality. It presumes that there is no _a priori_ knowledge of the number of cuttings to be harvested each year. There is, however, _ex ante_ knowledge of the form of production function for each cutting. Knowledge of this kind is necessary so that changes in value product due to incorrect fertilizer use can be assessed.

A further assumption is retained. Future alfalfa prices can be predicted with certainty in the spring. The main object here is to simplify the analysis.[1] However this assumption may not be too unreal. Table 15 is derived from the alfalfa hay prices of Appendix A. The table expresses the June, July and August prices as percentages of the prices ruling in the previous April.[2] There is a general downward

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[1]In Chapter VIII price is assumed absolutely uncertain.

[2]About which time fertilizer use decisions are made.

Table 15. Alfalfa hay prices for the months of June, July and August, 1944-1958, expressed as a percentage of the April price

Year	April	June	July	August
1944	100	86	84	86
1945	100	89	85	81
1946	100	96	95	100
1947	100	98	87	92
1948	100	81	104	106
1949	100	86	80	83
1950	100	91	81	88
1951	100	93	81	83
1952	100	86	90	105
1953	100	88	93	94
1954	100	88	85	89
1955	100	88	83	82
1956	100	112	115	116
1957	100	82	82	82
1958	100	87	85	83

trend in the June, July and August prices when compared with the April figure. Records are available for 15 years. In only one year (1956) does the June price fluctuate more than 10 percent round 90 percent of the April price. For July and August this is true for two and three years out of the 15, respectively. The months June, July and August are considered important. The reason is that in these months hay is normally bought in and sold. The preoccupation with hay prices arises because a change in price requires different fertilization rates if profits are to be maximized.

B. Variations in Expected Income Due to Uncertainty Surrounding the Number of Cuttings

The dilemma facing the decision maker has two aspects. The number of cuttings expected may be greater than the number realized. Therefore more fertilizer is applied than is necessary to maximize profits. Or, the number of cuts harvested may be greater than the number planned for. So not enough fertilizer has been applied. Consequently, profits obtained are less than could have been realized.

With regard to the alfalfa data, six possible outcomes are as follows:

Too much fertilizer may be applied.

(a) Two cuttings expected - one obtained.

(b) Three cuttings expected - one obtained.

(c) Three cuttings expected - two obtained.

Alternatively, too little fertilizer may be used.

(d) One cutting expected - two obtained.

(e) One cutting expected - three obtained.

(f) Two cuttings expected - three obtained.

The deviations from expected profits can be worked out for these situations. Take, for example case (a). The alfalfa yields for different fertilizer mixtures and rates are derived from production functions 4.1 and 4.4. A range of hay prices is then assumed ($15, $20 and $25 per ton).

The price of fertilizer is known. Thus net returns can be tabulated for different rates of fertilizer application. Consequently the profit maximizing rate becomes apparent. In example (a) two cuttings are expected, but one obtained. The amount of fertilizer maximizing returns for production function 4.4 is thus applied to returns conforming to production function 4.1. Using the prices assumed above, differences in net cash returns can be worked out. These differences can be regarded as gains or losses in net value product. The justification for this is that ex ante expectations are assumed the relevant ones in the mine of the decision maker. If another than anticipated value product accrues to him, his profits have been added to or reduced. If cases (b) to (f) are treated similarly, variations in net[1] returns can be tabulated in a similar fashion. Tables 16 to 21 correspond to situations (a) to (f).

The "no set ratio" mixture included in each table is the combination of P_2O_5 and K_2O which maximizes returns under the price conditions existing. Chapter V demonstrated that these amounts lead to greater net returns than any of the commonly used set-ratio mixtures.

Tables 16, 17 and 18 indicate by what extent net returns

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[1]Returns are "net" to the extent that the cost of fertilizer has been deducted.

Table 16. Overfertilization - reduction in expected net returns attributable to fertilizing in anticipation of two cuttings, only one cutting obtained

Mixture	Amount fertilizer applied (lbs./acre) P	K	Decline in expected net returns ($/acre) when hay price per ton is $15	$20	$25
No set ratio	31	67	1.85		
	54	77		3.26	
	67	82			4.57
0-20-20	0		0	0	0
	50		.36	.48	.60
	100		.86	1.16	1.44
	150		1.25	1.68	2.09
	200		1.63	2.18	2.72
	250[a]		1.99	2.66	3.32
	300[b]		2.32	3.10	3.87
	350[c]		2.64	3.52	4.40
0-35-15	0		0	0	0
	50		.53	.72	.89
	100		1.03	1.38	1.72
	150[a]		1.48	1.98	2.47
	200[b]		1.90	2.54	3.17
	250[c]		2.29	3.06	3.82
0-12-36	0		0	0	0
	50		.52	.70	.87
	100		.99	1.32	1.65
	150		1.40	1.88	2.34
	200[a]		1.75	2.34	2.92
	250[b,c]		2.05	2.74	3.42

[a]Quantity maximizing net returns for two cuttings when hay price $15 per ton.

[b]Quantity maximizing net returns for two cuttings when hay price $20 per ton.

[c]Quantity maximizing net returns for two cuttings when hay price $25 per ton.

Table 17. Overfertilization - reduction in expected net returns attributable to fertilizing in anticipation of three cuttings, only one cutting obtained

Mixture	Amount fertilizer applied (lbs./acre) P	K	Decline in expected net returns ($/acre) when hay price per ton is $15	$20	$25
No set ratio	45	72	4.68		
	64	79		7.24	
	76	83			9.67
0-20-20	50		.95	1.62	1.58
	100		1.96	2.62	3.27
	150		2.82	3.76	4.70
	200		3.58	4.78	5.97
	250		4.29	5.72	7.15
	300[a]		4.88	6.50	8.13
	350[b]		5.42	7.22	9.03
	400[c]		5.87	7.82	9.78
0-35-15	50		1.20	1.60	2.00
	100		2.28	3.04	3.80
	150		3.24	4.32	5.40
	200[a]		4.08	5.44	6.80
	250[b]		4.83	6.44	8.05
	300[c]		5.44	7.26	9.07
0-12-36	50		1.26	1.68	2.10
	100		2.33	3.10	3.88
	150		3.21	4.28	5.35
	200[a]		3.87	5.16	6.45
	250[b,c]		4.35	5.80	7.25

[a]Quantity maximizing net returns for three cuttings when hay price $15 per ton.

[b]Quantity maximizing net returns for three cuttings when hay price $20 per ton.

[c]Quantity maximizing net returns for three cuttings when hay price $25 per ton.

Table 18. Overfertilization - reduction in expected net returns attributable to fertilizing in anticipation of three cuttings - only two cuttings obtained

Mixture	Amount fertilizer applied (lbs./acre) P	K	Decline in expected net returns ($/acre) when hay price per ton is $15	$20	$25
No set ratio	45	72	2.47		
	64	79		3.68	
	76	83			4.83
0-20-20	50		.59	1.14	.98
	100		1.10	1.46	1.83
	150		1.57	2.08	2.61
	200		1.95	2.60	3.25
	250		2.30	3.06	3.83
	300[a]		2.56	3.40	4.26
	350[b]		2.78	3.70	4.63
	400[c]		2.93	3.90	4.88
0-35-15	50		.67	.88	1.11
	100		1.25	1.66	2.08
	150		1.76	2.34	2.93
	200[a]		2.18	2.90	3.63
	250[b]		2.54	3.38	4.23
	300[c]		2.82	3.76	4.70
0-12-36	50		.74	.98	1.23
	100		1.34	1.78	2.23
	150		1.81	2.40	3.01
	200[a]		2.12	2.82	3.53
	250[b,c]		2.30	3.06	3.83

[a]Quantity maximizing net returns for three cuttings when hay price $15 per ton.

[b]Quantity maximizing net returns for three cuttings when hay price $20 per ton.

[c]Quantity maximizing net returns for three cuttings when hay price $25 per ton.

Table 19. Underfertilization - addition to expected net returns when fertilizing in anticipation of one cutting but two cuttings obtained

Mixture	Amount fertilizer applied (lbs./acre) P	K	Addition to expected net returns ($/acre) when hay price per ton is $15	$20	$25
No set ratio	9	58	1.24		
	35	67		2.60	
	50	73			3.87
0-20-20	50		.36	.48	.60
	100		.86	1.16	1.44
	150[a]		1.25	1.68	2.09
	200		1.63	2.18	2.72
	250[b]		1.99	2.66	3.32
	300[c]		2.32	3.10	3.87
0-35-15	50		.53	.72	.89
	100[a]		1.03	1.38	1.72
	150[b]		1.48	1.98	2.47
	200[c]		1.90	2.54	3.17
0-12-36	50		.52	.70	.87
	100		.99	1.32	1.65
	150[a]		1.40	1.88	2.34
	200[b]		1.75	2.34	2.92
	250[c]		2.05	2.74	3.42

[a]Quantity maximizing net returns for one cutting when hay price $15 per ton.

[b]Quantity maximizing net returns for one cutting when hay price $20 per ton.

[c]Quantity maximizing net returns for one cutting when hay price $25 per ton.

Table 20. Underfertilization - addition to expected net returns when fertilizing in anticipation of one cutting but three cuttings obtained

Mixture	Amount fertilizer applied (lbs./acre) P	K	Addition to expected net returns ($/acre) when hay price per ton is $15	$20	$25
No set ratio	9	58	4.88		
	35	67		4.56	
	50	63			4.42
0-20-20	50		.95	1.62	1.58
	100		1.96	2.62	3.27
	150[a]		2.82	3.76	4.70
	200		3.58	4.78	5.97
	250[b]		4.29	5.72	7.15
	300[c]		4.88	6.50	8.13
0-35-15	50		1.20	1.60	2.00
	100[a]		2.28	3.04	3.80
	150[b]		3.24	4.32	5.40
	200[c]		4.08	5.44	6.80
0-12-36	50		1.26	1.68	2.10
	100		2.33	3.10	3.88
	150[a]		3.21	4.28	5.35
	200[b]		3.87	5.16	6.45
	250[c]		4.35	5.80	7.25

[a]Quantity maximizing net returns for one cutting when hay price $15 per ton.

[b]Quantity maximizing net returns for one cutting when hay price $20 per ton.

[c]Quantity maximizing net returns for one cutting when hay price $25 per ton.

Table 21. Underfertilization - addition to expected net returns when fertilizing in anticipation of two cuttings but three cuttings obtained

Mixture	Amount fertilizer applied (lbs./acre) P	K	Addition to expected net returns ($/acre) when hay price per ton is $15	$20	$25
No set ratio	31	67	2.20		
	54	77		3.50	
	67	82			4.68
0-20-20	50		.59	1.14	.98
	100		1.10	1.46	1.83
	150		1.57	2.08	2.61
	200		1.95	2.60	3.25
	250[a]		2.30	3.06	3.83
	300[b]		2.56	3.40	4.26
	350[c]		2.78	3.70	4.63
0-35-15	50		.67	.88	1.11
	100		1.25	1.66	2.08
	150[a]		1.76	2.34	2.93
	200[b]		2.18	2.90	3.63
	250[c]		2.54	3.38	4.23
0-12-36	50		.74	.98	1.23
	100		1.34	1.78	2.23
	150		1.81	2.40	3.01
	200[a]		2.12	2.82	3.53
	250[b,c]		2.30	3.06	3.83

[a]Quantity maximizing net returns for two cuttings when hay price $15 per ton.

[b]Quantity maximizing net returns for two cuttings when hay price $20 per ton.

[c]Quantity maximizing net returns for two cuttings when hay price $25 per ton.

are reduced, when the number of cuttings is overestimated. The amount of fertilizer applied is the quantity which maximizes profits if the number of cuttings is correct. Returns are reduced in two ways. Costs are increased by spending money on more than the _ex post_ profit maximizing quantity of fertilizer.[1] In addition total value product is less due to the smaller crop yield.

When three cuttings are expected but only one obtained, anticipated net returns are reduced most (Table 17). The expected profit maximizing quantity of fertilizer (300 pounds of 0-20-20) is applied. But anticipated net value product falls by $4.48 per acre when the price of hay is $20 per ton. If the hay price rises to $25, a reduction in expectations of $9.78 per acre is sustained. Under these circumstances minimization of loss might be considered an objective. If there is uncertainty as to whether one or three cuttings are likely, application of 200 pounds of 0-12-36 reduces anticipated net value product least (by $3.87 per acre when the price of hay is $15 per ton). On the other hand, expectations may be correct and three cuttings may be obtained. In this latter instance 200 pounds of 0-12-36 fertilizer does not give greatest profit when compared with other combinations (Tables

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[1]If expectations of the number of cuttings are correct, the quantities of fertilizer annotated in the tables maximize profits. If expectations are incorrect, the amount of fertilizer applied no longer maximizes profits _ex post_.

8 and 12).

The expected net value product is reduced least when the crop is fertilized in anticipation of two cuttings but only one is obtained. Table 16 outlines this situation. If 67 pounds of P_2O_5 and 82 pounds of K_2O are applied per acre, expected net returns are reduced by only $4.57 (when hay is selling for $25 per ton). But when three cuttings are expected and only one obtained the reduction in anticipated net returns is $9.67 per acre (Table 17).

Table 18 outlines the possibility which is closest to reality. Three cuttings are expected, fertilizer is applied accordingly, but only two cuts are harvested. When hay sells at $20 per ton, the reduction in anticipated net returns varies from $3.06 to $3.70 per acre depending on the mixture used. As hay prices rise, the decline in expected net returns grows correspondingly larger. When the hay price goes from $15 to $25 per ton the reduction in expected returns is approximately doubled. Thus when 300 pounds of 0-20-20 fertilizer is applied per acre, expected net returns decline by $2.56 or $4.26 depending on whether hay is selling at $15 or $25 per ton.

Tables 19, 20 and 21 relate to the situations in which expectations are too conservative. The number of cuttings obtained are greater than anticipated. Quantities of fertilizer which were (subjectively) presumed sufficient to maximize profits are less than required. The largest addition to anticipated net returns occurs when one cutting is expected but three are harvested (Table 20). Here, if hay is selling at $20 per ton, "windfall" returns amount to $5.72 per acre if 250 pounds of 0-20-20 is used. Or, an addition of $8.13 per acre to anticipated profits is possible if 300 pounds of 0-20-20 is used. The hay price in this latter case is $25 per ton.

If one cutting is expected but two are obtained (Table 19) the increase in expected value product is smallest. Nevertheless even when hay is only $15 per ton, the addition to expected net returns is between $1.03 and $1.40 per acre. The difference depends on the type of fertilizer applied. As the hay price rises the addition to anticipated net returns becomes of greater significance. When hay is selling at $25 per ton the increase in anticipated returns is at least half as great again than when the price of hay is $15 per ton.

The inference may be drawn from Tables 19-21 that profits are always increased by being conservative in estimating the number of cuttings. Likewise Tables 17-19 infer that overconfidence is always penalized. When hay is $25 per ton, conservatism is rewarded by an addition to anticipated net value product of \$8.13 per acre (Table 20). Optimism is penalized by reduction in expected net returns of up to \$9.78 per acre (Table 17). The amount of money is too large to neglect. However the basis on which this analysis is made should be recalled. Changes in profits indicated in Tables 16-21 are based on expectations which are incorrect ex post. It has been assumed that the farmer has a definite net profit figure in mind when making fertilizer use decisions. If his anticipations of the number of cuttings prove incorrect then this net profit figure is altered. Therefore his expected net returns are decreased or added to.

In addition, no account has been taken of the probability of getting one, two or three cuttings per year. Suppose the probability of getting one cut per year is low, and that of getting three cuts is high. Then, over a period of years, greater net returns may be realized by following the latter course every year. Variations in expected net returns of the magnitude indicated in Tables 17 and 20 may be large. But their significance depends on the frequency with which these variations are likely to occur over the planning period. This aspect is discussed in the following section.

C. Reduction of Uncertainty Due to the Probability Distribution of the Number of Cuts Being Known

Myers (18) has done work on the probabilities of runs of consecutive dry days at Corydon in southcentral Iowa. He took a "dry" day as being one of less than 0.2 inches of rainfall. Then he estimated the chances of the middle day in a five day period being involved in runs of 5, 10, 15, 20, 25, 30 and longer dry days in length.

Most farmers in southern Iowa take the first cut of alfalfa about the 15th of June, the second cut around the 25th of July and the third cut no later than the 1st of September. It is assumed that a four to five week dry period starting towards the end of June would result in only one cutting being taken. A three to four week dry period starting around the middle of July would preclude a third cut. Table 22 (adapted from Myers' data) indicates that in one year out of 20 only one cutting can be expected. In two years out of 10 a third cutting is unlikely.

The extent of the economic horizon now becomes of relevance. The 1954 Census of Agriculture (21) shows that the average length of time the Iowa farm occupier (tenant or owner) has been on his present farm is 13 years. The planning period for fertilizer use decisions is probably considerably less than this. For tenant operators this is especially true. A five year horizon is assumed. An added proviso is that in each of four years three cuttings are obtained. In the remaining year only two are harvested. The probability of getting only one cutting in the five year period is neglected.

Table 22. Probabilities of runs of consecutive dry days for five-day increments at Corydon, Iowa

Period	Number of dry days[a]							
	0	5	10	15	20	25	30	35
May 10-16	82	69	38	19	9			
	80	61	35	17	8			
	80	58	32	15	8			
	79	56	31	13	5			
	80	59	35	15	8			
June 14-20	82	65	40	20	10			
	84	68	43	26	15	7	5	
	84	68	44	29	19	10	6	
	85	68	46	31	21	13	8	5
	86	69	50	34	22	14	10	6
July 19-25	88	78	57	39	25	14	9	5
	88	79	55	39	24	14	9	5
	84	70	50	34	21	14	8	5
	84	68	48	31	20	14	8	5
	85	70	52	35	23	14	10	6

[a]Probability of a length of run greater than the number of days indicated.

Under these circumstances two possible courses of action are:

(a) The alfalfa is fertilized in expectation of three cuttings every year.

(b) In one out of the five years, fertilizer is applied at the rate which maximizes returns if two cuttings are obtained. *Ex post* this decision is correct or incorrect. If the latter is true, it is further assumed that in one year only two cuttings are obtained when three are expected.

At the end of a five year period the net returns situation based on ex ante expectations, conforms to one of the possibilities outlined in Table 23.[1] Profit maximizing quantities of fertilizer for the various situations were obtained from Tables 6, 7, 8, 10, 11 and 12. Hay yields and net returns were then worked out. Hay prices were taken as $15, $20 and $25 per ton.

Situation A is the case in which anticipations prove correct over the whole five year period. Common mixtures may be used in contrast to specific amounts. Use of the former may result in a reduction of income of up to $12.20 per acre over the whole period (when the price of hay is $25 and 0-35-15 is used rather than the no set ratio mixture). Of the mixes 0-20-20 gives the greatest net returns if hay prices are high. If hay is selling for $25 per ton, use of 0-20-20 rather than 0-35-15 results in extra returns of $11.97 over the period. Over one year this amounts to $2.39, which is still a substantial difference.

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[1]As case A is the only one in which expectations are wholly correct, net returns here should be highest. Table 23 does not confirm this belief. The reason is because fertilizer is applied in 50 pound increments. For maximum net returns using a particular mixture, 229 pounds per acre may be necessary when one cutting is obtained. If two cuttings are realized 253 pounds may be needed to maximize profits. For three cuttings 269 pounds may maximize returns. But the tables are drawn up so that it is possible that the profit maximizing amount appears as 250 pounds in each case.

Table 23. Net returns due to fertilizing over a five year period assuming various methods of fertilization

Situation	Mixture used	Increased net returns due to fertilization when hay price per ton is $15	$20	$25
A. For 4 years expects 3 cuts, gets 3 For 1 year expects 2 cuts, gets 2	No set ratio	25.50	48.95	74.75
	0-20-20	24.21	48.30	74.52
	0-35-15	17.75	39.20	62.55
	0-12-36	24.38	44.34	64.82
B. For 4 years expects 3 cuts, gets 3 For 1 year expects 3 cuts, gets 2	No set ratio	25.38	48.87	74.67
	0-20-20	24.09	48.30	74.52
	0-35-15	17.57	39.12	62.55
	0-12-36	24.38	44.34	64.82
C. For 3 years expects 3 cuts, gets 3 For 1 year expects 2 cuts, gets 3 For 1 year expects 3 cuts, gets 2	No set ratio	25.23	48.75	74.60
	0-20-20	23.95	47.60	74.27
	0-35-15	15.33	38.72	63.38
	0-12-36	24.38	43.60	64.82

The decision may be made to disregard the probability of getting only two cuts in one of the five years. The assumption may be that three cuttings can be expected every year. Situation B gives the net returns when this course is followed. Compared with case A, returns are reduced slightly. In the most extreme case the reduction is only 18 cents per acre (when 0-35-15 is used and hay is selling at $15 per ton). Moreover this reduction occurs over a five year period.

In case C expectations prove correct in three years out of the five. But net value product is not reduced by a large amount. The most unfavorable example is when 0-35-15 is used throughout the period and hay is priced at $15 per ton. Net profits fall by $2.42 per acre compared with the 0-35-15 situation in case A. When 0-20-20 is used and hay is selling at $20 per ton net profits are reduced by 70 cents, compared with case A. But over one year this reduction only amounts to 14 cents. Use of different mixtures, therefore, may or may not have a sizeable effect on net profits.

Summarizing, there are two factors influencing net profits:

(a) The choice of a particular fertilizer mixture.

(b) The amount of fertilizer applied when expectations of the number of cuttings prove incorrect.

For this study factor (a) has more influence on net returns.

The basic assumption of this chapter has been that only once in a five year planning period will three cuttings of alfalfa not be harvested. The probability of getting only one cut in any one year has been rejected altogether. Uncertainty still remains as to the actual year in which two cuttings are obtained. For the data of this study it has been shown that decreases in net income due to incorrect fertilizer use decisions can be minimized by assuming that three cuttings will always be obtained. At least, this conclusion holds for the situations examined. The reduction in net income by acting as though three cuttings will always be obtained amounts to 4 cents per year when measured against correct anticipation of situation A in Table 23. This loss is small enough to disregard. The same does not apply to differences in net returns arising from use of different fertilizer mixtures. These differences are such that meaningful recommendations can still be made concerning the type of mixture to use.

VII. THE UTILIZATION PROBLEM

A. Introduction

Previous analysis (Chapters V and VI) has assumed that the alfalfa crop is harvested as hay. However this is not always true. The present chapter is concerned with profit maximizing fertilization rates when alfalfa is utilized in a form other than hay. It is assumed that there is <u>ex ante</u> knowledge of future alfalfa use. Thus there is no problem of allocating fertilizer resources among ends which are uncertain.

There are many alternative avenues of utilization of alfalfa. Attention is confined to these possibilities:

(a) Used to make hay.

(b) Fed green-chopped to dairy cows.

(c) Used as summer pasture for pigs.

In order to derive economic optima a value must be assigned to the crop. In the case of alfalfa used as hay this has been taken as the local market price (Chapter V). However alfalfa is not usually sold green-chopped or as pasture. So there is no ruling price in the latter two instances. The crop may be used for dairy cow or pig feeding, thus replacing other feedstuffs. Hence it assumes a value equal to the cost of the feeds it substitutes for. This concept of replacement value is somewhat naive. There are many factors other than cost to be taken into consideration when deciding whether or not to feed alfalfa in place of other concentrates. Nevertheless the replacement value approach is useful in demonstrating one method of deriving optimum fertilization rates. Once a value has been put on a crop fertilizer may be applied so that value product is maximized.

Appendices B and C detail the procedures by which replacement values are estimated. The values are on a hay-equivalent basis so that comparisons are facilitated.

The quantities of fertilizer which maximize profits when alfalfa is made into hay have already been discussed in Chapter V. The remainder of the present chapter deals with fertilization rates when alfalfa is fed either green-chopped to dairy cows or as pasture to pigs.

B. Alfalfa Fed Green-chopped to Dairy Cows

As outlined in Appendix B the basic green alfalfa ration is 150 pounds of green alfalfa plus 5 pounds of corn and cob meal (Ration 1). (The rations are on a per cow, per day basis.) Ration 1 is assumed to replace Ration 2 or Ration 3. Ration 2 consists of 35 pounds of hay plus 11 pounds of meal while Ration 3 is 15 pounds of hay plus 25 pounds of meal. Depending on the prices assumed for hay and meal in the rations, values can be assigned to the green-chopped alfalfa.

If the green alfalfa is expressed in terms of its equivalent weight of alfalfa hay some resulting prices per ton are as follows:

Meal and hay prices	Low	Medium	High
Value of green alfalfa used in place of Ration 2	$16.20	$21.40	$26.60
Value of green alfalfa used in place of Ration 3	$17.80	$22.80	$28.00

The table is read from left to right. For example the basic green alfalfa ration (Ration 1) may replace Ration 2. When hay and meal prices are low, the value of the green alfalfa is $16.20 per equivalent ton of hay. The prices represent the amount of money the decision maker is willing to pay for green chopped alfalfa. The assumption here is that all rations are equally preferable in other respects. As the alfalfa is an intermediate product, the farmer may be thought of as selling the alfalfa to himself. Acting as a seller he wishes to maximize value product. Hay and fertilizer prices are known so his problem is to fertilize to the stage where net profits are greatest.

As far as the present study is concerned profit maximizing quantities of fertilizer were worked out for the 1st+2nd+3rd cuttings. The procedure was as follows. Alfalfa yields were derived from the basic production function 4.5 for various levels of fertilization. Returns were then calculated using the hay-equivalent prices above. In the simplified situation assumed, the only cost was that of fertilizer. Deduction of fertilizer costs gave net value product figures. The amounts of fertilizer which maximized net value product were then apparent. The results of these calculations are shown in Tables 24 and 25. In Table 24 the alfalfa is valued as though it replaced Ration 2, while in Table 25 the alfalfa prices are based on the assumption that the alfalfa replaced Ration 3. In both tables it is apparent that net value product is maximized when the quantities of P_2O_5 and K_2O applied are "optimum" and not set-ratio mixtures. Unfortunately this kind of <u>ex post</u> knowledge is less than ideal when <u>ex ante</u> predictions are required. Another problem is that crop response in future years may not be similar. So the small differences in net value product apparent between the "no set ratio" portions and the rest of the tables may not warrant special recommendations. If 0-20-20 is used rather than a "no set ratio" mixture net returns may only be decreased by 22 cents (Table 24) or 15 cents (Table 25) per acre (at the lowest alfalfa price in each case). In Table 24 the greatest difference among optimum dressings amounts to \$2.56 per acre. This results when 79 pounds of P_2O_5 and 84 pounds of K_2O are applied per acre in contrast to 300 pounds of 0-35-15. The difference in Table 25 is slightly higher, because of the higher alfalfa

Table 24. Net value product from various fertilizer inputs when green chopped alfalfa used in place of Ration 2

Type fertilizer	Quantity (lbs./acre) P	K	Cost fertilizer ($/acre)	Yield alfalfa (tons/acre)	Value of alfalfa per acre (less fertilizer cost) when hay price per ton is $16.20	$21.40	$26.60
No set ratio	51	74	8.80	3.335	45.23		
	68	81	10.85	3.446		62.89	
	79	84	12.10	3.499			80.97
0-20-20	0		0	2.379	38.54	50.91	63.28
	50		1.50	2.588	40.42	53.88	67.34
	100		3.00	2.777	41.99	56.43	70.87
	150		4.50	2.946	43.22	58.54	73.86
	200		6.00	3.095	44.14	60.23	76.33
	250		7.50	3.225	44.74	61.51	78.28
	300		9.00	3.334	45.01	62.35	79.68
	350		10.50	3.424	44.97	62.77	80.58
	400		12.00	3.494	44.60	62.77	80.94
0-35-15	0		0	2.379	38.54	50.91	63.28
	50		2.12	2.636	40.58	54.29	68.00
	100		4.25	2.860	42.08	56.95	71.83
	150		6.37	3.051	43.06	58.92	74.79
	200		8.50	3.209	43.48	60.17	76.86
	250		10.62	3.335	43.41	60.75	78.09
	300		12.75	3.427	42.77	60.59	78.41
	350		14.87	3.487	41.62	59.75	77.88
0-12-36	0		0	2.379	38.54	50.91	63.28
	50		1.50	2.623	40.99	54.63	68.27
	100		3.00	2.830	42.85	57.36	72.28
	150		4.50	3.000	44.10	59.70	75.30
	200		6.00	3.132	44.74	61.02	77.31
	250		7.50	3.228	44.79	61.58	78.36
	300		9.00	3.286	44.23	61.32	78.41
	350		10.50	3.308	43.09	60.29	77.49

Table 25. Net value product from various fertilizer inputs when green chopped alfalfa used in place of Ration 3

Type fertilizer	Quantity (lbs./acre) P	K	Cost fertilizer ($/acre)	Yield alfalfa (tons/acre)	Value of alfalfa per acre (less fertilizer cost) when hay price per ton is $17.80	$22.80	$28.00
No set ratio	57	77	9.55	3.379	50.60		
	72	82	11.30	3.466		67.72	
	81	85	12.35	3.508			85.87
0-20-20	0		0	2.379	42.35	54.24	66.61
	50		1.50	2.588	44.57	57.51	70.96
	100		3.00	2.777	46.43	60.31	74.76
	150		4.50	2.946	47.94	62.67	77.99
	200		6.00	3.095	49.09	64.57	80.66
	250		7.50	3.225	49.90	66.03	82.80
	300		9.00	3.334	50.34	67.01	84.35
	350		10.50	3.424	50.45	67.57	85.37
	400		12.00	3.494	50.19	67.66	85.83
	450		13.50	3.545	49.60	67.33	85.76
0-35-15	0		0	2.379	42.35	54.24	66.61
	50		2.12	2.636	44.80	57.98	71.69
	100		4.25	2.860	46.66	60.96	75.83
	150		6.37	3.051	47.94	63.19	79.06
	200		8.50	3.209	48.62	64.66	81.35
	250		10.62	3.335	48.74	65.42	82.76
	300		12.75	3.427	48.25	65.38	83.21
	350		14.87	3.487	47.20	64.63	82.77
0-12-36	0		0	2.379	42.35	54.24	66.61
	50		1.50	2.623	45.19	58.30	71.94
	100		3.00	2.830	47.37	61.52	76.24
	150		4.50	3.000	48.90	63.90	79.50
	200		6.00	3.132	49.75	65.41	81.70
	250		7.50	3.228	49.96	66.10	82.88
	300		9.00	3.286	49.49	65.92	83.01
	350		10.50	3.308	48.38	64.92	82.12

values. It amounts to \$2.86 per acre when 300 pounds of 0-12-36 are used rather than 81 pounds of P_2O_5 and 85 pounds of K_2O.

As Ration 3 costs more than Ration 2 the replacement value of the alfalfa is higher in the former case. This means that optimum rates of fertilization are also higher. Thus an application of 250 pounds per acre of 0-20-20 maximizes net returns when the hay-equivalent price if \$16.20 per ton (Table 24). But 350 pounds are required when the price is \$17.80 (Table 25). Among the mixtures, 0-20-20 gives the highest net value product. The increase in value product through using 0-20-20 becomes more apparent as the price of the alfalfa rises.

Although both tables confirm that common mixes are inferior to individual amounts, the difference may not always be significant. This situation may arise if the _ex post_ production function for a particular year proves quite different from the one used as a basis for recommendations. If environmental conditions differ markedly from year to year production functions will also differ. Alternatively, predictions may be made on the basis of insufficient data.

C. Alfalfa Used as Pasture for Pigs

It may be considered that the best method of utilization of the alfalfa pasture is as a grazing supplement for fattening pigs. Appendix C outlines a procedure for valuing alfalfa used in this fashion. The availability of alfalfa pasture means that the level of concentrates (corn and cob and soybean meals) fed under drylot conditions is lowered.

When the series of low, medium and high prices shown in Appendix C is assumed for corn and cob meal and soybean meal the replacement value per equivalent hay ton of alfalfa pasture is $17.45, $21.44 and $25.43, respectively. These are the prices that the farmer acting as a seller may expect. As in the previous section, by use of the basic production function 4.5, alfalfa yields can be estimated for various rates of fertilizer application. Fertilizer and hay prices are also known. Thus profit maximizing quantities of fertilizer can be indicated. The results are included in Table 26. Discussion is confined to the first plus second plus third cuttings.

The table indicates that use of common fertilizer mixes does not give the greatest net returns. If the replacement value of the alfalfa as hay is $21.44 per ton, net value product is maximized by application of 68 pounds of P_2O_5 and 81 pounds of K_2O per acre. However, use of 350 pounds of 0-20-20 results in a crop only .022 tons per acre lighter. But if either 0-12-36 or 0-35-15 fertilizer is used, the yields and consequent net value products are less favorable

Table 26. Net value product for various fertilizer inputs when pigs fed alfalfa pasture and Ration 5 in place of Ration 4 (drylot)

Type fertilizer	Quantity (lbs./acre) P	K	Cost fertilizer ($/acre)	Yield alfalfa (tons/acre)	Value of alfalfa per acre (less fertilizer cost) when hay price per ton is $17.45	$21.44	$25.43
No set ratio	56	76	9.40	3.371	49.42		
	68	81	10.85	3.446		63.03	
	77	83	11.85	3.489			76.87
0-20-20	0		0	2.379	41.51	51.01	60.50
	50		1.50	2.588	43.66	53.99	64.31
	100		3.00	2.777	45.46	56.54	67.62
	150		4.50	2.946	46.91	58.66	70.42
	200		6.00	3.095	48.01	60.36	72.70
	250		7.50	3.225	48.78	61.64	74.51
	300		9.00	3.334	49.18	62.48	75.78
	350		10.50	3.424	49.25	62.91	76.57
	400		12.00	3.494	48.97	62.91	76.85
	450		13.50	3.545	48.36	62.50	76.65
0-35-15	0		0	2.379	41.51	51.01	60.50
	50		2.12	2.636	43.88	54.39	64.91
	100		4.25	2.860	45.66	57.07	68.48
	150		6.37	3.051	46.87	59.04	71.22
	200		8.50	3.209	47.50	60.30	73.10
	250		10.62	3.335	47.58	60.88	74.19
	300		12.75	3.427	47.05	60.72	74.40
	350		14.87	3.487	45.98	59.87	73.80
0-12-36	0		0	2.379	41.51	51.01	60.50
	50		1.50	2.623	44.27	54.74	65.20
	100		3.00	2.830	46.38	57.67	68.97
	150		4.50	3.000	47.85	59.82	71.79
	200		6.00	3.132	48.65	61.15	73.65
	250		7.50	3.228	48.83	61.71	74.59
	300		9.00	3.286	48.34	61.45	74.56
	350		10.50	3.308	47.22	60.42	73.62

still. Assume the replacement value of the alfalfa expressed in terms of hay is \$25.43 per ton. Use of 250 pounds of 0-12-36 or 300 pounds of 0-35-15 results in a decrease in net value product of \$2.26 and \$2.45 per acre, respectively, when compared with the net returns from using 400 pounds of 0-20-20.

This chapter has assumed *a priori* knowledge of crop use and of the form of the production function. It was shown that with this knowledge fertilizer use decisions could be made which minimized the cost of producing a required output. In the experiment under consideration, of the fertilizer mixtures, 0-20-20 gave greatest net returns. But because in practice the exact form of the production function is not known, recommendations to farmers may not always be correct.

VIII. UTILIZATION AND NUMBER OF CUTTINGS UNCERTAIN

A. Introduction

In each succeeding chapter the analysis has moved closer to reality. Chapter V dealt with optimum fertilization rates, under the assumption that the number of cuttings expected was known. Chapter VI withdrew this simplifying premise. In Chapter VII the amount of fertilizer recommended was shown to vary depending on the use of the alfalfa. The present chapter deals with the more complex problem of levels of fertilization when both the cuttings expected and the utilization of the crop are unknown, at the time when fertilizer is applied. Uncertainty as to the former is readily accounted for by fluctuating weather conditions. The assumption that there is no _ex ante_ knowledge regarding the use of the alfalfa is also justifiable. As a dry summer progresses hay prices may become favorable relative to feed prices. Hence alfalfa may be harvested for hay rather than fed green chopped to dairy cows. The market for hogs may be depressed, so that production costs may be lowered by using the alfalfa pasture as a run off. Machinery used for forage harvesting may break down so the crop may be turned into hay rather than left to go to stalk.

As discussed in Chapter VII a series of values can be imputed to the alfalfa depending on its use. In a general sense, the value to the farmer may be regarded as the price he would otherwise pay for substitute feedstuffs. If hay is required, but not made, it must be bought in. If green alfalfa is not used as part of a dairy cow ration, its place must be taken by other feeds (whose costs are usually more easily determined). While alfalfa hay is an input with a consequent cost to a cattle raiser, it may also be a saleable product for a person who farms less intensively.

The simplification that is made in this chapter is that the farmer who grows alfalfa as an intermediate product regards its replacement value (Chapter VII) or its sale value as a price. This is the price he is willing to pay (himself, in effect) for use of the crop in a further stage of the production process. The fertilization problem thus becomes one of price uncertainty. The price is dependent on the imputed value. However the value is itself unknown, as this is based on the use to which the alfalfa is put. At the time of fertilization, use of the alfalfa is uncertain.

This is not the usual profit maximizing situation in which resources are used so that marginal value product is maximized. If the latter were the case then the use commanding the highest price would always be chosen. An intermediate product is being dealt with. Long run profit maximization in the dairy or hog enterprise may require that the crop be fed in a particular form. Dairy cows may need green chopped alfalfa, not only to maintain milk production but to keep in good condition. The alfalfa hay may even give a higher net return, if sold, than the increased milk production attributable to the green alfalfa.

The other facet of this problem - that of uncertainty concerning the number of cuttings - has been resolved in Chapter VI. There it was shown that the decision maker could always act as though three cuttings were certain. It is emphasized that this solution is relevant only under the particular circumstances of the experiment under discussion.

B. Decision Making Under Uncertainty

If the analysis is continued in the intermediate product context, the situation is that alfalfa is being grown for sale for various purposes. A series of prices is known, corresponding to the various uses. But, as utilization is uncertain, there is no <u>ex ante</u> knowledge of which price will be realized. If the alfalfa is kept, or sold as hay, its price is \$15, \$20 or \$25 per ton, depending on the state of the market. Price is no longer assumed certain as in Chapter VI. When the crop is fed green-chopped to dairy cows, the price per equivalent ton of hay is \$16.20, \$21.40 or \$26.60 (Appendix B). As pasture for pigs the price is \$17.45, \$21.44 or \$25.43 per ton (Appendix C). The price which is relevant out of each set depends on whether prices for corn and cob meal and soybean meal are low, average or high. It is assumed that when there is a low hay price, feed prices are also low. There is a similar correspondence among average and high prices.

The problem of level of fertilization now becomes one of decision making under absolute uncertainty,[1] sometimes known as a game against nature (13). Game theory involves problems of conflicts of interest. A number of players are required to make a choice from a well defined set of choices. The gain or loss of one person depends not only on his own actions but on the actions of other persons. The problem for each player is what choice should he make in order that his partial influence over the outcome benefits him most. The outcomes may be expressed as elements of a matrix.

In games against nature a matrix is given. A player must choose a row. A column will be chosen by "nature", a fictitious player having no known objective and no known strategy. As far as this study is concerned, the farmer must choose from among a set of acts, A_1, A_2 ... A_m, but the relative desirability of each act depends upon which state of nature prevails (either s_1, s_2 ... s_n). To each pair (A_i, s_j) consisting of an act and a state there is a consequence or

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[1]Absolute uncertainty means that a series of prices is known, but which price is relevant is unknown.

outcome. For the alfalfa fertilization situation, the game matrix is presented in Table 27.

In the table, the method of utilization corresponds to the acts and the states of nature to the price levels. Each

Table 27. Game matrix for alfalfa fertilization problem

Act	State of nature		
	s_1	s_2	s_3
A_1. Sells or keeps as hay	15.00	20.00	25.00
A_2. Feeds green chopped to dairy cows	16.20	21.40	26.60
A_3. Feeds as pasture to pigs	17.45	21.44	25.43

price is the utility associated with the consequence of the pair (A_i, s_j). The problem has now been put in a game theory context. There are a number of possible states of nature, as well as a number of possible actions that the farmer can take. He does not know which state of nature is the true one. So he still has the problem of deciding which course to follow. The decision concerning the action to take may be based on certain criteria. These have been commonly used in the past in an attempt to resolve the decision problem under uncertainty. The criteria select the act which is optimal according to the particular criterion used. The remainder of this chapter is concerned with the examination and application of these criteria.

1. <u>The maximin criterion</u>

This has been suggested by Wald (22). Each act is appraised by looking at the worst state for that act. The optimal choice is the one with the best worst state. In order to apply this criterion each act is assigned its security level as an index. The security level is the least amount receivable from any strategy. For Table 27 the index for act A_1 is 15.00, for act A_2 16.20 and for act A_3 17.45. The act whose security index is a maximum is A_3. Therefore according to the maximin criterion the farmer should fertilize the alfalfa in expectation of feeding it to pigs. The criterion is conservative, in that relative to each act it concentrates on the one having the worst consequence. However a generally accepted value judgment is that farmers <u>are</u> conservative. Therefore for this study this criterion may be more acceptable than some.

2. <u>The pessimism-optimism index criterion</u>

Hurwitz (11) first formulated this criterion. It is a less conservative method of decision making. A judgment is formed based on a weighted combination of the best and worst states. The best and worst states are weighted according to a pessimism-optimism index. Compilation of this index requires another value judgment being made. This relates to whether farmers are pessimistic or optimistic.

For act A_i let m_i be the minimum and M_i the maximum of the $s_{i1}, s_{i2} \cdots s_{in}$. A fixed number α between 0 and 1 called the pessimism-optimism index is chosen. With each A_i the index $\alpha m_i + (1 - \alpha)M_i$ is associated. Of two acts the one with the higher α index is chosen. If farmers are considered conservative, α might be taken as being between .5 and .8. In Table 28 α indices for values of α ranging from

Table 28. Pessimism-optimism index criterion, αm_i and $(1 - \alpha)M_i$ values

		αm_i	$(1 - \alpha)M_i$	$\alpha m_i + (1 - \alpha)M_i$
$\alpha = .3$	A_1	4.50	17.50	22.00
$1 - \alpha = .7$	A_2	4.86	18.62	23.48
	A_3	5.23	17.80	23.03
$\alpha = .4$	A_1	6.00	15.00	21.00
$1 - \alpha = .6$	A_2	6.48	15.96	22.44
	A_3	6.98	15.26	22.24
$\alpha = .5$	A_1	7.50	12.50	20.00
$1 - \alpha = .5$	A_2	8.10	13.30	21.40
	A_3	8.72	12.71	21.43
$\alpha = .6$	A_1	9.00	10.00	19.00
$1 - \alpha = .4$	A_2	9.72	10.64	20.36
	A_3	10.47	10.17	20.64
$\alpha = .7$	A_1	10.50	7.50	18.00
$1 - \alpha = .3$	A_2	11.34	7.98	19.32
	A_3	12.21	7.63	19.84
$\alpha = .8$	A_1	12.00	5.00	17.00
$1 - \alpha = .2$	A_2	12.96	5.32	18.28
	A_3	13.96	5.09	19.05

.3 to .8 are included. For the α values .5 to .8 the index shows that act A_3 is optimal. In terms of the study this means that the farmer should fertilize in anticipation of feeding the alfalfa to pigs.

3. Principle of insufficient reason criterion

The principle was first systematized by Jacob Bernoulli (1654-1745) (13). It states that if there is no evidence showing that one event from an exhaustive set of mutually exclusive events is more likely to occur than another, then the events should be judged equally probable. As far as game theory is concerned this principle is usually associated with the name of Laplace (16).

For the fertilizer problem it is assumed that in Table 27 there is complete ignorance as to which state among s_1, s_2 ... s_n obtains. Behaviour should therefore be based on the assumption that they are all equally likely. The situation then becomes one of risk with a uniform probability distribution over all the states. In order to decide what course to follow each act is assigned an index, as follows:

$$\frac{s_{i1} + s_{i2} + \cdots s_{in}}{n}$$

The act with the largest index is chosen. For Table 27 act A_1 has an index of 20.00, act A_2 of 21.40 and act A_3 of 21.44. Once more the farmer should act as though it was certain that the alfalfa would be used as pasture for pigs.

Preferences for acts, according to each criterion, are as follows:

Wald	A_3 over A_2 over A_1
Hurwitz	A_3 over A_2 over A_1
Laplace	A_3 over A_2 over A_1

Act A_3 is considered optimal as far as all these criteria are concerned.

Through use of the criteria a decision has been made. However there may be some doubt as to whether the farmer decision maker does, in fact, apply similar reasoning in solving everyday problems. J. L. Dillon (4) examined farmers' solutions to a set of hypothetical decision problems and concluded that the majority of farmers tended to use an approach either of the Wald or Laplace type.

Now while this application of game theory has indicated which act is considered optimal (but not necessarily true, ex post) the expected price remains uncertain. Thus there is still doubt as to the optimum quantity of fertilizer to use. The changes in value product due to applying other than the profit maximizing quantity of fertilizer, are examined in the next section.

C. Consequences of Incorrect Decision Making

Assuming that the decision to apply fertilizer in expectation that the crop will be used as pasture for pigs is correct, ex post, the value of the pasture is still absolutely uncertain ex ante. Thus the amount of fertilizer applied may not maximize net value product. Variations in net value product when alfalfa replacement values or hay prices change, are shown in Table 29. The figures represent the increase in value product (less fertilizer cost) due to applying fertilizer at all.

If act A_3 is true (i.e. is the ex post outcome) and 0-20-20 fertilizer is applied, 350 pounds maximizes net returns when the alfalfa[1] is valued at $17.45 or $21.44 per ton. If the replacement value of the alfalfa is $25.43 per ton then 400 pounds of 0-20-20 is optimum. In the latter case, use of only 350 pounds decreases value product by 28 cents per acre. But if 400 pounds is applied when only 350 pounds maximizes profits, the decline in net value product is 28 cents per acre for the $17.45 price or zero for the $21.44 price.

Use of 0-20-20 fertilizer gives a greater net value product than either 0-35-15 or 0-12-36, but the latter two may be used for one reason or another. In the case of 0-12-36, 250 pounds per acre always gives maximum net returns whatever the alfalfa replacement value. For 0-35-15, use of 250 pounds maximizes net value product except when the

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[1]More precisely, the equivalent weight of alfalfa expressed as hay.

Table 29. Net value product of alfalfa due to fertilization when prices vary

Type of mixture	(lbs./ acre)	Act A_1 Used as hay, Hay price per ton			Act A_2 Used green chopped, Replacement value per ton			Act A_3 Used as pasture, Replacement value per ton		
		$15	$20	$25	$16.20	$21.40	$26.60	$17.45	$21.44	$25.43
0-20-20	250	5.19	9.42	13.65	6.20	10.60	15.00	7.27	10.63	14.01
	300	5.33	10.10	14.88	6.47	11.44	16.40	7.67	11.47	15.28
	350	5.18	10.40	15.63	6.43	11.86	17.30	7.74	11.90	16.07
	400	4.73	10.30	15.88	6.06	11.86	17.66	7.46	11.90	16.35
	450	3.99	9.82	15.63	5.39	11.45	17.52	6.85	11.49	16.15
0-35-15	150	3.71	7.07	10.43	4.52	8.01	11.51	5.36	8.03	10.72
	200	3.95	8.10	12.25	4.94	9.26	13.58	5.99	9.29	12.60
	250	3.72	8.50	13.28	4.87	9.84	14.81	6.07	9.87	13.69
	300	2.97	8.21	13.45	4.23	9.68	15.13	5.54	9.71	13.90
	350	1.75	7.29	12.83	3.08	8.84	14.60	4.47	8.86	13.30
0-12-36	150	4.82	7.92	11.03	5.56	8.79	12.02	6.34	8.81	11.79
	200	5.30	9.06	12.83	6.20	10.11	14.03	7.14	10.14	13.15
	250	5.24	9.48	13.73	6.25	10.69	15.08	7.32	10.70	14.09
	300	4.61	9.14	13.68	5.69	10.41	15.13	6.83	10.44	14.06
	350	3.44	8.08	12.73	4.55	9.38	14.21	5.71	9.41	13.12

alfalfa value is \$25.43 per ton. Here the decrease in returns due to using 250 pounds per acre rather than the profit maximizing quantity of 300 pounds is 21 cents per acre.

If act A_3 is true application of the optimal amount of 0-20-20 fertilizer gives a net value product of \$7.74, \$11.90 or \$16.35 per acre depending on the replacement value of the alfalfa. On the other hand if 0-35-15 fertilizer is used, maximum net returns are \$6.07, \$9.87 or \$13.90 per acre. Thus use of one mixture rather than another may result in a reduction of net returns of \$1.67, \$2.03 or \$2.45 per acre. It is concluded that variations in net value product due to using different fertilizer mixtures are greater than changes in net returns attributable to incorrect decision making with respect to use of a single fertilizer. The conclusion applies to fertilizer use when the expected method of utilization is actually realized.

If the decision to fertilize for pig pasture is incorrect <u>ex post</u> variations in anticipated and realized net returns may be greater as there is a wider spread in possible levels of fertilization for acts A_1 and A_2. If act A_3 is expected to be true and 0-20-20 fertilizer applied, there are only two possible levels of fertilization which maximize profits. These are 350 or 400 pounds per acre. The one which is relevant depends on the ruling replacement value for the alfalfa. For acts A_1 and A_2 the optimum rates of fertilization using the 0-20-20 mixture are 300, 350 or 400 pounds per acre, depending on the alfalfa values. When 0-35-15 fertilizer is used the optimum amounts to apply if act A_3 is true are 250 or 300 pounds per acre subject to the ruling alfalfa value. Or if acts A_1 or A_2 are true the profit maximizing amounts of fertilizer to apply are either 200, 250 or 300 pounds per acre contingent on the alfalfa values. If 0-12-36 fertilizer is used in expectation that act A_3 is true a single rate of 200 pounds per acre is optimum whatever the alfalfa value. By contrast if act A_1 or A_2 is true the profit maximizing amount of fertilizer to apply is at one of two levels according to the alfalfa replacement values.

The significance of applying other than the optimum amount of fertilizer is related to the price of the product, and consequently influences net returns. For example, 400 pounds of 0-20-20 fertilizer may be used in expectation of an act A_3 alfalfa price of \$25.43 per ton when <u>ex post</u> act A_2 with a \$26.60 alfalfa price is true. Thus there is a gain in net value product of \$1.31 per acre. On the other hand if act A_1 eventuates (when the alfalfa replacement value is \$25 per ton) expected net value product per acre falls by \$.47.

When 0-20-20 is applied 350 pounds per acre may be considered optimum for act A_3. The decision maker may expect a net return of \$7.74, \$11.90 or \$16.07 per acre depending on the alfalfa values. But if act A_1 is true the difference between expectations (act A_3 value products) and realizations (act A_1 value products) is \$2.56, \$1.50 or \$.44 per acre. If the alfalfa is used green chopped act A_3 expectations differ from act A_2 realizations by \$1.31, \$.04 or \$1.23 according to the alfalfa replacement values.

These variations assume less serious proportions if the decision maker has no definite expected net returns figure in mind, but assesses <u>ex post</u> returns against the <u>ex post</u> optimum. Assume that act A_3 is considered true <u>ex ante</u>. Therefore the supposedly profit maximizing quantity of 350 pounds of 0-20-20 fertilizer is applied. But act A_1 is found to be true <u>ex post</u>. For act A_1 350 pounds of fertilizer only maximizes profits when the hay price is \$20 per ton. If hay is selling at \$15 per ton then 50 pounds too much fertilizer has been applied per acre. Potential value product is reduced by 15 cents per acre. On the other hand when hay is priced at \$25 per ton the optimum fertilization rate is 400 pounds per acre. By using only 350 pounds value product is decreased by 25 cents per acre. Alternatively act A_2 may be true <u>ex post</u> but the profit maximizing amount of 0-20-20 fertilizer for act A_3 is applied (350 pounds per acre). The difference between actual and potential net returns is 4 cents, zero or 36 cents per acre depending on the alfalfa replacement values.

The conclusion may be drawn that variations in net returns are not large when other than optimum quantities of a particular fertilizer mixture are used. However the changes in value product by using profit maximizing quantities of one fertilizer mixture rather than another have yet to be examined. These variations may be of greater magnitude.

Assume that for some reason the 0-35-15 mixture is used rather than 0-20-20 fertilizer. Act A_3 value product maximizing quantities of fertilizer are applied in each case. The reduction in net value product per acre due to using 0-35-15 fertilizer rather than 0-20-20 is as follows:

If act A_1 is true $1.46, $1.90 or $2.35

If act A_2 is true $1.56, $2.02 or $2.49

The actual reduction will depend on the alfalfa replacement price. The figures above correspond to the low, medium or high hay or hay-equivalent prices in the relevant part of Table 29.

Alternatively if 0-12-36 fertilizer is used rather than 0-20-20 the reduction in net value product per acre is as follows:

If act A_1 is true +$.06, $.92 or $1.90

If act A_2 is true $.18, $1.19 or $2.22

It is apparent that these differences are large compared with the variations in profit arising from the use of a non-optimum quantity of a single fertilizer. Thus choice of a particular fertilizer mixture can affect profits significantly.

The analysis in this chapter indicates that the fertilization problem may be treated as one of decision making under absolute uncertainty. But this gives no indication of the price which would determine the optimum amount of fertilizer to apply. However it has been shown that if the farmer acts as though act A_3 is correct and fertilizes accordingly, the differences in net value product should act A_1 or act A_2 prove true, ex post, are small. Of greater importance are variations in value product when one particular fertilizer mixture is used rather than another. In this respect for the experiment under consideration, 0-20-20 is recommended. Beyond stipulating the type and approximate quantity of fertilizer to use no further precision can be incorporated into recommendations.

IX. SUMMARY AND CONCLUSIONS

Any farmer decision maker who uses fertilizer on a crop is concerned with at least two problems relating to the amount of fertilizer which maximizes profits. First, crop yield is uncertain. Thus the quantity of fertilizer applied may not be optimum for the yield actually obtained. Second, at the time of planting, future crop use may also be indefinite. The value of the alfalfa may differ according to the method of utilization. Hence a particular amount of fertilizer applied in the spring may not maximize potential value product.

This study is concerned with these problems as they relate to the P_2O_5 and K_2O fertilization of an alfalfa crop, from which three cuttings were obtained throughout the growing season.

For yield estimation and derivation of economic optima, a quadratic type of function was considered to fit the data best. The function included linear, squared and interaction terms.

Initially, the analysis estimated optimum quantities of fertilizer to use under different capital and resource use situations. The number of cuttings expected per year was assumed known with certainty. Many farmers use mixtures already made up to set specifications (*e.g.*, 0-20-20). The three most common ones were included in the analysis to determine what influence their application had on net returns. It was found that there were significant variations in profits associated with the use of different mixtures and combinations.

Subsequently the analysis moved closer to reality by no longer assuming _a priori_ knowledge of the number of cuttings harvested each year. Climatic data was available concerning the probability of drought conditions in the area. It was concluded that over a five year planning period, three cuttings could be expected in four years and two in the remaining year. _Ex ante_ the decision maker anticipates the number of cuttings and applies fertilizer accordingly. His expectations may or may not be correct. It was shown that losses could be minimized by acting as though three cuttings could be expected every year.

A procedure was outlined whereby a value could be imputed to the alfalfa depending on its method of utilization. _Ex ante_ knowledge of the form of the production function was also assumed. Fertilizer mixtures which minimize the cost of producing a required output were then predicted.

The most complex situation analyzed was that in which both the number of cuttings and utilization were considered uncertain, at the time of application of the fertilizer.

This problem was treated as one of decision making under absolute uncertainty (a game against nature). Three decision making criteria (Wald, Hurwitz, Laplace) were then applied in order to determine what course the farmer should follow. Each criterion pointed to the same act as being optimal. Use of game theory did not guarantee that the act chosen was true *ex post*. In addition, price expectations were still uncertain for the act chosen. However it was shown for this particular set of data that if decisions were based on the criteria, variations in net returns were small even if price expectations proved incorrect.

A general conclusion of the study is that as far as profits are concerned, greatest importance attaches to the use of a *particular* mixture (*e.g.*, 0-20-20 rather than 0-35-15). The fact that under conditions of yield and use uncertainty, 250 rather than 300 pounds of a mixture are used, does not affect profit to the same extent.

The data presented has shown that differences in net profits arising from use of various fertilizer mixtures and combinations are not always large. The criticism may arise that at times an unwarranted specious definiteness has been given to the analysis, particularly concerning the use of one mixture rather than another. The objection may be considered even more justifiable if the environment is judged at all variable. This is because the study presumes *ex ante* knowledge of the form of the fertilizer production function. These criticisms may be correct but it is maintained that further precision than is sometimes apparent can be incorporated into fertilizer use recommendations.

There are some dangers in using production function data for predictive purposes. Recommendations may be made on the basis of one experiment carried out under particular environmental conditions over a single year. But unless the circumstances are similar in future years, rates of fertilization suggested may not maximize profit. If however data is built up for various soil types under changeable environmental conditions, greater accuracy can be achieved in advice to farmers.

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XII. APPENDIX A. MID-MONTH PRICES RECEIVED BY IOWA FARMERS FOR ALFALFA HAY AT LOCAL MARKETS

Year[1]	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1944	21.00	20.50	20.20	20.00	20.30	17.30	16.90	17.30	17.00	17.50	18.40	20.00
1945	21.50	22.00	21.20	21.10	19.50	18.70	18.00	17.00	16.70	16.80	17.00	17.00
1946	18.00	18.00	18.10	17.30	16.60	16.60	16.50	17.30	17.30	18.30	18.80	21.10
1947	20.30	19.00	19.60	19.00	19.00	18.30	16.50	17.50	20.00	20.00	22.00	26.00
1948	26.00	23.50	23.70	24.00	24.00	23.60	25.00	25.50	27.00	27.50	26.30	27.00
1949	28.40	28.00	27.30	26.00	25.00	21.00	20.50	21.50	20.00	21.50	22.20	22.50
1950	22.30	22.20	21.50	21.50	21.50	18.50	17.50	19.00	17.50	18.00	18.10	20.00
1951	20.00	21.00	19.50	20.10	19.40	18.30	16.20	16.70	16.80	18.40	18.30	20.70
1952	20.40	19.90	19.90	18.80	18.70	17.40	16.90	19.70	21.20	21.60	22.20	22.10
1953	23.40	23.00	21.50	20.20	20.50	17.30	18.70	19.00	20.30	21.90	22.50	24.00
1954	24.00	21.70	21.70	21.60	20.00	19.00	18.30	19.30	20.30	19.80	20.40	21.00
1955	21.00	21.00	20.50	19.00	18.00	16.70	15.70	15.60	17.50	17.00	18.00	18.20
1956	19.00	18.00	17.80	17.80	21.00	20.00	20.40	20.70	20.00	17.70	20.50	22.10
1957	21.20	20.30	20.20	19.20	18.60	15.80	15.70	15.70	14.80	15.70	15.30	16.60
1958	16.60	15.20	15.00	14.90	13.90	13.00	12.60	12.30	12.80	13.10		

[1]Source: Iowa Crop and Livestock Reporting Service, Des Moines, Iowa

XIII. APPENDIX B. PROCEDURE FOR VALUING ALFALFA FED GREEN CHOPPED TO DAIRY COWS[1]

Assume 1400 lb. Holstein cow producing 50 lbs. 3.5% milk per day.

	T.D.N. (lbs.)	D.P. (lbs.)
Cow needs for maintenance (per day)	10.60	.87
for milk production	15.00	2.30
	25.60	3.17

	T.D.N. (lbs.)	D.P. (lbs.)
Fresh cut alfalfa 100 lbs. contains	14.70	3.40
Alfalfa hay 100 lbs. contains	50.30	10.50
Corn and cob (3 parts) and oat (1 part) meal 100 lbs. contains	72.42	6.32
Soybean meal 100 lbs. contains	78.90	38.10

1 lb. fresh green cut alfalfa is equivalent to .28 lbs. hay.
1 lb. alfalfa hay is equivalent to 3.57 lbs. green alfalfa.

Meal ration

3 parts corn and cob meal
1 part oats

If price of corn per bushel (56 lbs.) is $1.10
If price of oats per bushel (32 lbs.) is $ .60

If 70 lbs. of ear corn yields 1 bushel shelled corn.
Then under these conditions 1 lb. of meal mixture costs 1.64 cents.

Three levels of hay and meal prices are assumed:

	Meal price (cents/lb.)	Hay price (cents/lb.)
Low	1.30	.75
Medium	1.65	1.00
High	2.00	1.25

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[1]Feed requirements adapted from Morrison (17).

Three rations are proposed as follows:

Ration 1. All green alfalfa cow will eat

	T.D.N. (lbs.)	D.P. (lbs.)
Requirements	25.60	3.17
150 lbs. green alfalfa (equivalent to 42 lbs. hay) provide	22.05	5.10

So needs 3.55 lbs. T.D.N.

This is provided by 4.90 lbs. corn and oat meal (say 5 lbs.).

Ration 2. All the alfalfa hay the cow will eat (35 lbs. per day)

	T.D.N. (lbs.)	D.P. (lbs.)
Requirements	25.60	3.17
35 lbs. alfalfa hay	17.60	3.67

Needs 8.00 lbs. T.D.N.

So requires 11.04 lbs. meal (say 11 lbs.).

Ration 3. 15 lbs. alfalfa hay per day

	T.D.N. (lbs.)	D.P. (lbs.)
Requirements	25.60	3.17
15 lbs. alfalfa hay	7.54	1.57

Needs 18.06 lbs. T.D.N. and 1.60 lbs. D.P.

So requires 24.94 lbs. meal (say 25 lbs.)

If the farmer does not feed green alfalfa but uses either Ration 2 or Ration 3 the replacement value of the alfalfa may be computed as follows:

Ration 2	Cost of ration when meal and hay prices (cents) are Low	Medium	High
Meal requirement 11 lbs.	14.30	18.15	22.00
Hay requirement 35 lbs.	26.25	35.00	43.75
Total cost	40.55	53.15	65.75
Less meal cost of ration 1	6.50	8.25	10.00
Replacement cost of alfalfa	34.05	44.90	55.75
150 lbs. green alfalfa equivalent to 42 lbs. alfalfa hay			
Thus value per lb. as hay	.81	1.07	1.33
Hay price per ton	$16.20	$21.40	$26.60

Ration 3	Low	Medium	High
Meal requirement 25 lbs.	32.50	41.25	50.00
Hay requirement 15 lbs.	11.25	15.00	18.75
Total cost	43.75	56.25	68.75
Less meal cost of ration 1	6.50	8.25	10.00
Replacement cost of alfalfa	37.25	48.00	58.75
Value per lb. as hay	.89	1.14	1.40
Hay price per ton	$17.80	$22.80	$28.00

XIV. APPENDIX C. PROCEDURE FOR VALUING ALFALFA PASTURE FED TO PIGS[1]

Assume that the pigs are fed for 100 days (from 60 lbs. to market weight).

Method of feeding	Corn and cob meal (lbs./day)	Soybean meal (lbs./day)
Dry lot (Ration 4)	5.1	.48
Run on pasture (Ration 5)	4.0	.34
	1.1	.14

In 100 days the pasture replaces:

110 lbs. corn meal 14 lbs. soybean meal

Assume 8 pigs run per acre

Total feed replaced:

880 lbs. corn meal 112 lbs. soybean meal

Assuming the following corn and soybean meal prices:

	Corn and cob meal ($/bushel)	Soybean meal ($/100 lbs.)
Low	.80	2.80
Medium	1.00	3.20
High	1.20	3.60

This gives a per acre value to the alfalfa as follows:

Concentrate price level	Value of alfalfa ($/acre)
Low	15.70
Medium	19.28
High	22.87

Now,

1 lb. corn and cob meal contains
.7242 lbs. T.D.N., .0632 lbs. D.P.
1.1 lbs. corn and cob meal contains
.7966 lbs. T.D.N., .0695 lbs. D.P.

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[1]Feed requirements adapted from Morrison (17).

1 lb. soybean meal contains
.7890 lbs. T.D.N., .3810 lbs. D.P.
.14 lbs. soybean meal contains
.1105 lbs. T.D.N., .0533 lbs. D.P.

So one pig requires:

.9071 lbs. T.D.N. and .1228 lbs. D.P.

1 lb. fresh alfalfa contains .147 lbs. T.D.N. and .03 lbs. D.P.

So require 6.17 lbs. fresh alfalfa per day.

Assuming a 30% spoilage and wastage rate,

Daily requirement 8.02 lbs.

So eight pigs for 100 days require 6416 lbs. fresh alfalfa

Now 1 lb. hay = 3.57 lbs. alfalfa.

Thus hay equivalent required is 1797 lbs. hay.

Thus value of hay equivalent per ton:

With meal prices low	$17.45
With meal prices medium	$21.44
With meal prices high	$25.43